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# State Estimation Using Non-uniform and Delayed Information: A Review

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**Abstract:** The study and application of methods for incorporating nonuniform and delayed information in state estimation techniques are important topics to advance in soft sensor development. Therefore, this paper presents a review of these methods and proposes a taxonomy that allows a faster selection of state estimator in this type of applications. The classification is performed according to the type of estimator, method, and used tool. Finally, using the proposed taxonomy, some applications reported in the literature are described.

**Keywords:** State estimation, asynchronism, delayed measurement, non-uniform information, taxonomy.

## 1 Introduction

In almost all industrial processes, there is a need to carry out control, diagnostics, fault detection, identification, and monitoring[1, 2]. In modern industries, many variables need to be measured to achieve optimal automation and computing. However, in some cases, this is an arduous and expensive task due to the unavailability of reliable devices, time delays, errors in the measurement system, high cost of devices, and a hostile environment for primary measuring[3]. In order to overcome many of the above issues, state estimators are used to make estimates through measurements of other variables related to the hard-to-measure variables. In industrial applications, state estimators are implemented as software routines in dedicated hardware, usually known as soft or virtual sensors.

In this respect, continuous research of state estimation techniques allows applications in areas such as electrical and electromechanical systems, aeronautical and navigation systems, robotics, and recently in chemical and biotechnological processes. A recent paper proposed a classification of observers applied in chemical processes[4]. This classification is composed of six classes based on the review of current applications in specific chemical process systems. The classes are: Luenberger-based observers, finite-dimensional system observers, Bayesian estimators, disturbance and fault detection observers, artificial intelligence (AI)-based observers and hybrid observers. Another recent research paper presents a tutorial on the main Gaussian filters and estimation[5]. In that paper, the main Gaussian filters are explained in detail, considering linear optimal filtering, nonlinear filtering, adaptive filtering, and robust filtering. In addition, the authors describe a 200-year history of the main classical contributions to estimation theory. Finally, the authors highlight some trends such as the adaptive Kalman filter, the adaptive filter with parameter tuning, the adaptive filter with joint estimation of states and parameters, multiple models adaptive filtering and variable structure filtering and its variants. However, both classifications do not consider problems such as delay and multi-rate associated with available information from sensors or off-line analysis equipment. This kind of information is available in several processes but it is not commonly used despite allowing for an improvement in the quality of the estimation. Additionally, a recent review of multi-sensor distributed fusion estimation (DFE) taking into account data quantization, random transmission delays, packet dropouts and fading measurements is described in [6]. The proposed classification was based on some DFE algorithms in the literature and in the analysis of the phenomena of sensor networks. However, in this classification, the algorithms are only limited to the Kalman filter and its modifications. In addition, the phenomena of sensors network are not taken into account within the different applications of the estimation techniques. Therefore, a taxonomic classification that considers the phenomenon of acquisition, storage and use of non-uniform and delayed information of real applications is necessary. In addition, a classification that relates stochastic and deterministic estimation techniques is important.

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Nonuniform and delayed information produces collateral problems in state estimation techniques: multi-sampling, asynchronism, data loss, and variability in the degree of reliability of the information (precision or accuracy). For the handling of such information, specialized methods are required. In this regard, some authors have developed different methods based on stochastic estimation techniques[7−12]. These methods are arranged into two types: measurement fusion and augmented state space methods. Methods based on measurement fusion are only suitable for discrete-time systems. Those methods are designed for the Kalman filter and its variants. In contrast, the methods based on augmented state retain the original state-space representation of the process, making it more promising to facilitate their extension to different types of estimators. Furthermore, the conservation of the state space representation allows for the subsequent analysis of convergence, observability, and robustness of the estimator.

Some authors present deterministic estimation techniques with asynchronous and delayed measurements for hybrid systems, with a continuous-time model for the process and a discrete-time model for the effects of sensor and sampling. Such observers are arranged into four types: piece-wise observers[13], cascade observers[14,15], distributed observers[16], and partial state observers[17−20]. These deterministic estimation techniques allow the solution of problems presented in the state estimation independently or in stages, i.e., the adaptation of a hybrid estimator according to the needs of the system. Some partial stages may be signal processing, data prediction or estimation of unknown parameters.

Although state estimation with asynchronous and delayed information is a subject of current research, to the best of our knowledge, there is no review paper summarizing and collecting the whole spectrum of different estimation techniques, its limitations, tools, and applications. Some papers work on applied specific problems and a few show state estimation in bioprocess with delay measurement[21−23]. Moreover, there is no unified conceptual framework and taxonomy tools that enable researchers in this field to use and identify appropriate tools for their particular problems.

Therefore, in this paper, a review of the main methods and concepts for processes with non-uniform and delayed information is described. Additionally, a taxonomic organization of the reported methods is proposed. This taxonomic organization allows for faster selection of a state estimator incorporating non-uniform and delayed information. Finally, a comprehensive list of applications of state estimators in different processes is shown.

The paper is organized as follows. In Section 2, a framework to address the use of non-uniform and delayed information on estimation and control tasks is proposed. Section 3 presents the main methods for using non-uniform and delayed information on state estimation techniques and explains the proposed taxonomy for these methods. In Section 4, some applications reported in the literature are presented and analyzed. Finally, conclusions are summarized.

## 2 Nonuniform and delayed information in industrial processes

### 2.1 Basic definitions

In industrial processes, a large amount of information is stored in a supervisory control and data acquisition (SCADA) system. To handle the gathered information for state estimation and process control tasks, it is necessary to characterize and identify their sources and associated problems. For the taxonomy presented in current work, Definitions 1–4 are required:

**Definition 1.** Information is all symbolic representation of an event. This representation has meaning to whoever receives those symbols and helps him to interpret the world and to reduce the uncertainty.

**Definition 2.** A source of information is any origin of information as previously defined.

**Definition 3.** Delay is the time lapse that a signal takes from its source in a process until its reception at a storage or processing place.

**Definition 4.** Multi-sampling or asynchronism is the effect that occurs when the sampling time between two signals is not the same. In industrial processes, this effect occurs by the difference in the time response and delay of sensor technology.

### 2.2 Sources of information

Sources of information can be classified according to the characteristics of acquisition and storage in two types. In the first type, called on-line, there are on-line sensors connected to the SCADA system for measuring simple and common variables like level, temperature, flow, pressure, etc. In the second type, called off-line, more complex variables are obtained from samples taken from the process which are processed in the laboratory or by specialized equipment to obtain the variable value. A common example of off-line variables are the analysis variables like concentration. Values of off-line variables are stored in the SCADA system with a pre-stated time interval. Although on-line measurement is the best option, some off-line variables must be used due to cost or unavailability of on-line sensors for some variables required for process analysis and control.

A representation of the sources of information is shown in Fig. 1. From Fig. 1, it can be seen that the variables of a process may be inputs, outputs and states. Each variable may be known (measured) or unknown (unmeasured). In the diagram, the measured inputs are marked as $u$, the unmeasured inputs as $d$, the measured states as $x_a$ and $x_b$, the unmeasured states as $x_c$, and the measured outputs as $y$ and $y_\theta$. Such information may be derived from two different sources: on-line measurements $y$ or off-line measurements $y_\theta$. For on-line measurements, the state vector $x_a$ is measured by sensors, transformed into $y$ and immediately stored in the SCADA system. In the case of off-line measurements, a sample of the process is taken to determine the state vector $x_b$, then the sample is analyzed in the laboratory or specialized equipment to get the output $y_\theta$. The laboratory or specialized equipment result is manually stored in the SCADA system. For this reason, each information source is collected and stored with particular characteristics, including sampling time, delay and degree of reliability (accuracy and precision), among others.

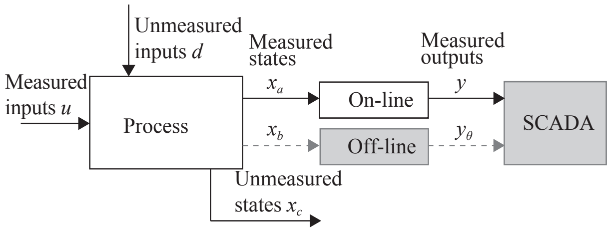


Fig. 1 Sources of information in an industrial process

## 2.3 Acquisition and storage of information

In a SCADA system, all information is stored at discrete times, according to Assumptions 1–4[8, 14]:

**Assumption 1.** Sampling delays associated with on-line measurements are considered negligible compared to the sampling delays associated with off-line measurements.

**Assumption 2.** All the measurements available at the time-instant $k$ are dated. That is, the subset of measurements corresponding to the uniform and non-uniform class of measurements is known exactly. In addition, the value of delay is known for each measurement.

**Assumption 3.** The information obtained from the uniform measurements is more susceptible to problems of noise and precision than that obtained from laboratory or specialized equipment analysis. The first one is subject to the characteristics of the signal conditioning system of the sensors. In the second, strict adherence to high-quality standards is assumed, even if measurements are non-uniform.

**Assumption 4.** Off-line information is subject to human error while storing it into the SCADA system. Errors can be represented as spurious or missing data.

A characterization of phenomena occurring in the acquisition and storage of data from each source of information is presented in Fig. 2. In Fig. 2, the lower horizontal line represents the time instant at which the sampling is performed. Moreover, the upper horizontal line represents the time instant at which measurements are acquired and stored in the SCADA system. The straight-dashed vertical lines represent the on-line measurements obtained by the sensors. Note that these measurements are sampled with a fixed sampling period $T$ and their measurement delays are considered negligible.

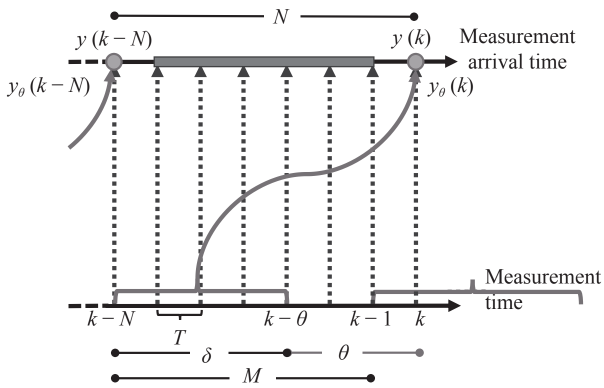


Fig. 2 Characterization of phenomena occurring in the acquisition and storage of information sources. Source: modified from [8].

Off-line measurements, obtained from the analysis of samples in laboratory or specialized equipment, are represented with a continuous and curved line. It is worth mentioning that off-line measurements may have different measurement delays $\theta$. This means that the elapsed time since the sample is taken until the measurement arrives to the SCADA system, denoted by $N$, where $N = \delta + \theta$ can be a time-varying parameter. $M$ represents the time period between two successive off-line samples. According to the characteristics of the sources of information (on-line and off-line), the information can be uniform, non-uniform or integral.

**Definition 5.** A uniform (synchronous and undelayed) measurement is a measurement that is available at every constant period of time. See straight and dashed line in Fig. 2.

**Definition 6.** A non-uniform measurement is a measurement that is not necessarily available at every constant period of time. In addition, once the measurement is available, the obtained information is related with old system trajectories. See curved and continuous lines in Fig. 2.

**Definition 7.** An integral measure is an off-line measurement characteristic in which a measurement value can be effective or valid over a given time period $\delta$. The integral measurements define that "the delayed measurements can also be a function of the integral of the states over a certain past period of time"[8].

It must be clarified that the term nonuniform information does not refer to the type of distribution or a statistical characteristic of the data. In this regard, and according to the above definitions, the process information is considered non-uniform when the sampling time or the delay time is different between physical measurements.

On the other hand, the information storage process can have two instances: major and minor, defined as:

**Definition 8.** A minor instance case concerns the case where only uniform measurements are available at a given time-instant $k$.

**Definition 9.** A major instance case concerns the case where all measurements (uniform and non-uniform) are available at a given time-instant $k$.

### 2.4 Mathematical preliminaries

An industrial process in presence of non-uniform and delayed information can be represented as a discrete-time non-linear system of the form:

$$x(k+1) = f(x(k), u(k), \varepsilon(k)) \tag{1}$$

where $x \in \mathbf{R}^n$ is the system state, $u \in \mathbf{R}^m$ is the system input and $\varepsilon$ is the process noise. The function $f$ represents the non-linear dynamics of the system. Finally, it is assumed that both uniform and non-uniform measurements exist, denoted here by $y(k) \in \mathbf{R}^{r_1}$ and $y_\theta(k) \in \mathbf{R}^{r_2}$, respectively.

In practical applications, two cases concerning the system outputs can be available at every time-instant $k$:

Minor instance

$$\begin{aligned} y(k) &= h_1(x(k), v^1(k)) \\ y_\theta(k) &= [\,] \end{aligned} \tag{2}$$

Major instance

$$\begin{aligned} y(k) &= h_1(x(k), v^1(k)) \\ y_\theta(k) &= h_2(x(k), v^2(k)) \end{aligned} \tag{3}$$

where the subscript $(\cdot)_\theta$ denotes non-uniform information, the symbol $[\,]$ denotes an empty vector, and the discrete-time $(k-\theta)$ represents a known, and possibly varying, delayed time-instant. $h_1$ and $h_2$ are non-linear functions. The vector $v^i$ with $i = 1, 2$ stands for noise for uniform and nonuniform measurement respectively.

The estimation problem concerns the use of model (1) and the available measurements (2) and (3) to find an estimation of the system state $x$, denoted $\hat{x}$, in such a way that a norm of the estimation error $e = x - \hat{x}$ will be minimized. In a compact form, the problem is:

**Problem 1.** Given a system model (1): How to incorporate non-uniform measurements into the state estimation process under Assumptions 1 to 4, and considering both minor and major instances in a common framework?

In this section, the basic concepts were defined and the phenomenon of the acquisition and storage of industrial information sources was characterized. According to the characteristics of the information sources mentioned, their use in state estimation techniques is not a trivial matter. Collateral problems can occur such as: multi-sampling or asynchrony, missing and spurious data and redundant information, among others[8, 9]. Therefore, in the next section, a review of methods to use non-uniform and delayed information in state estimation techniques is presented.

## 3 Methods to use non-uniform and delayed information in state estimation techniques

From the literature, it is possible to identify different tools developed from the information and control theory to manage and incorporate non-uniform and delayed information in state estimation techniques. In this paper, two types of systems will be discussed: stochastic and deterministic systems.

### 3.1 State estimation in stochastic systems with delayed measurements

Several methods have been proposed for state estimation when the plant is modeled by a discrete-time stochastic model. In this case, the models consist of a deterministic part and a stochastic component characterized by the mean and the variance in the measurement noise and model noise. Fig. 3 shows the main methods reported for state estimation techniques incorporating non-uniform and delayed information[7–12]. Below, a discussion of each method is presented.

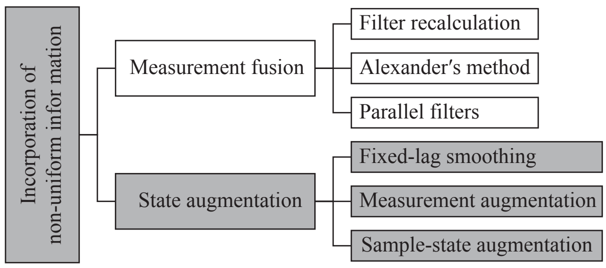


Fig. 3 Methods to incorporate non-uniform and delayed information in stochastic state estimation techniques

#### 3.1.1 Methods based on measurement fusion

These methods are developed for the use of multisensors or redundant measurements. For example, in [24], the position of a wheelchair is estimated using the fusion of two sources of information to improve the performance of the estimator. A source of measurement is obtained from a compass and an odometer, and the other from the same compass and an accelerometer.

In the literature, three variants of methods based on measurement fusion are presented: filter recalculation, Alexander′s method and parallel filter. These methods are based on the readjustment of the estimate in the major instance. In filter recalculation, the readjustment of the current state is performed by recalculating the entire trajectory of the Kalman filter. The recalculation is performed from the sampling $(k-N)$ to the major instance

($k$) (see Fig. 4). In Fig. 4, $C$, $C_\theta$ and $K$ are uniform output matrix, non-uniform output matrix and the Kalman filter gain, respectively. $y_t$ refers to the total output of the system in the major instance, i.e., $y_t = [y \quad y_\theta]^{\mathrm{T}}$. In Alexander's method, each type of measurement (on-line and off-line) is treated statistically independently and separately[7]. Finally, the parallel filters are an extension of Alexander's method, which guarantees the optimization at all times of measurement fusion.

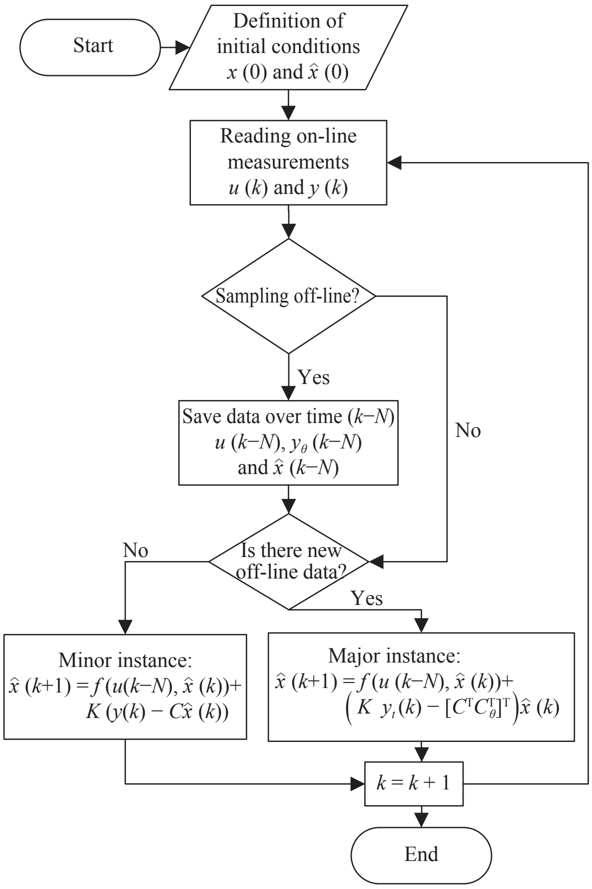


Fig. 4 Algorithm flowchart for measurement fusion methods

The methods based on measurement fusion only apply to discrete-time systems. These are developed to work with the Kalman filter and its variants[25, 26].

### 3.1.2 Methods based on state augmentation

The methods based on state augmentation are based on enlargement of the state space with information from off-line measurements and subsequently an extended model is incorporated into the state estimation technique, following the algorithm illustrated in Fig. 5. In Fig. 5, $G(\cdot)$ and $\hat{Z}(\cdot)$ represent the non-linear dynamic function of the estimated augmented state space and the augmented state vector, respectively. In state augmentation methods, the model dimension is augmented conserving the state-space representation. The methods of state augmentation have three variants reported in the literature[7]: fixed-lag smoothing, measurement augmentation and sample-state augmentation (see Fig. 3). In the fixed-lag smoothing method, the $N$ past states are smoothed using on-line measurements of the minor instances. Finally, when off-line and delayed measurements are obtained, both measurements (off-line and on-line) are used to smooth out the state between $(k-N)$ and $(k)$. However, the problem with the fixed-lag smoothing method is the computational cost generated by the high-order of the increased state space. To overcome this drawback, method variations like measurement augmentation and sampled-state augmentation have been developed. The measurement augmentation method is used when the number of off-line measurements $r_2$ is less than the size of the state $n$. The other variation (sampled-state augmentation) is justified by the fact that the delayed information is a function of only the state at the sampling time $x(k-N)$ and only this information must be retained until the major instance. The methods based on augmentation state retain the state space representation, allowing its extension to different types of estimators[27, 28], including deterministic estimation techniques.

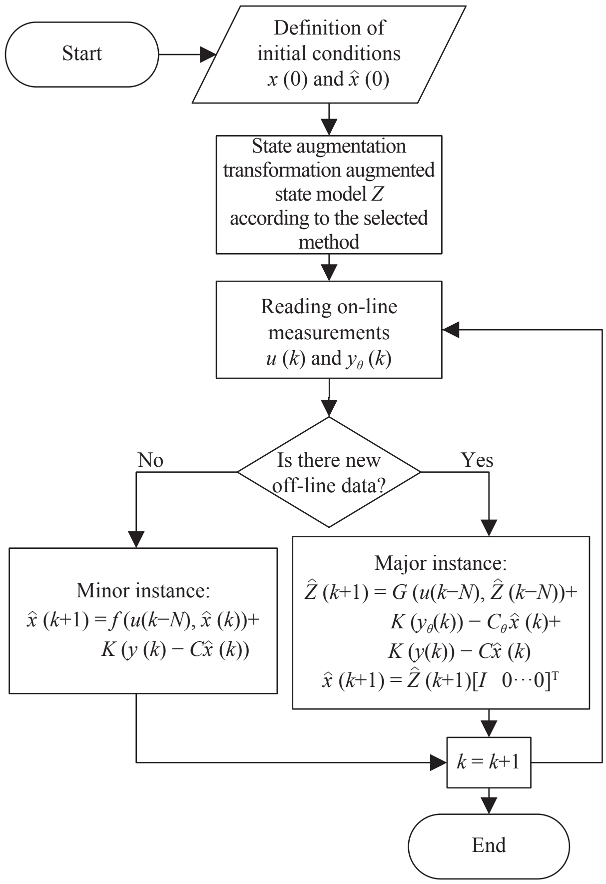


Fig. 5 Algorithm flowchart for state augmentation methods

### 3.1.3 Mathematical description of a linear system with augmented state

Due to the importance of augmented state methods, a brief mathematical description of augmented state representation for a linear system is presented based on [7]. System (1) can be represented as a discrete-time linear system as

$$x(k+1)=Ax(k)+Bu(k)+\varepsilon(k). \tag{4}$$

And according to the minors (2) and (3) instances for the discrete-time linear system (4), we have

Minor instance

$$\begin{aligned} &y(k)=Cx(k)+v^1(k)\\ &y_\theta(k)=[\,]. \end{aligned} \tag{5}$$

Major instance

$$\begin{aligned} &y(k)=Cx(k)+v^1(k)\\ &y_\theta(k)=C_\theta x(k-N)+v^2(k-N) \end{aligned} \tag{6}$$

where $x(k)\in\mathbf{R}^n$, $u(k)\in\mathbf{R}^m$, $y(k)\in\mathbf{R}^{r_1}$ and $y_\theta(k)\in\mathbf{R}^{r_2}$ are states, inputs, uniform outputs and non-uniform outputs of the system, respectively. $\varepsilon(k)$ and $v^i(k)$ correspond to model uncertainty and measurement noises. $A$, $B$, $C$ and $C_\theta$ are state, input and output matrices, respectively.

Now, by the concept of state augmentation, the order of the system can be increased such that the system contains the information of non-uniform and delayed measurements, as shown in (7).

$$\begin{aligned} &Z(k+1)=\Phi Z(k)+\Gamma U(k)+\Psi\varepsilon(k)\\ &Y(k)=\Xi Z(k)+v^1(k) \end{aligned} \tag{7}$$

where the matrices of the augmented state space $\Phi$, $\Gamma$, $\Psi$ and $\Xi$ are used instead of $A$, $B$, $Q$ and $C$, respectively. According to the augmented state method, the augmented state vector $Z$ and augmented matrices are defined in Table 1.

Note the high order of the augmented system produced by the augmentation methods of fixed-lag smoothing and measurement. This can lead to high computational cost when the estimation uses higher order and complex models[7].

Now, the above representation can be extended to the non-linear system (1). For this, the augmented state vector $Z$ can be redefined using the Jacobian of the non-linear functions $f$, $h_1$ and $h_2$ of system (1). The measurement instances (2) and (3) in this case are as follows:

$$\begin{aligned} &F=\left.\frac{\partial f}{\partial x}\right|_{(x(k),u(k))},F_u=\left.\frac{\partial f}{\partial u}\right|_{(x(k),u(k))}\\ &H_1=\left.\frac{\partial h_1}{\partial x}\right|_{(x(k),u(k))},H_2=\left.\frac{\partial h_2}{\partial x}\right|_{(x(k-N),u(k-N))} \end{aligned} \tag{8}$$

where $F$, $F_u$, $H_1$ and $H_1$ are the Jacobian matrices for system (1) and its measurements instances (2) and (3). Then using the fixed-lag smoothing method, the concept of state augmentation can be applied to redefine the matrix representation as follows:

$$Z(k)=[x(k)^{\rm T}x^{\rm T}(k-1)\cdots x^{\rm T}(k-N)] \tag{9}$$

$$\begin{aligned} &\Phi^*=\begin{bmatrix} F&0&\cdots&0&0\\ I&0&\cdots&0&0\\ 0&I&\cdots&0&0\\ \vdots&\ddots&\ddots&\ddots&\vdots\\ 0&0&\cdots&I&0 \end{bmatrix},\Gamma=\begin{bmatrix} F_u\\0\\0\\ \vdots\\0\end{bmatrix}\\ &\Xi^*=\begin{bmatrix} H_1&0&\cdots&0&0\\ 0&0&\cdots&0&H_2\end{bmatrix}. \end{aligned} \tag{10}$$

Table 1 Characteristics of state augmentation methods

| Method | State dimension | Augmented matrix transformation |
| --- | --- | --- |
| Fixed-lag smoothing | $N\times n+n$ | $Z(k)=[x^{\rm T}(k)\quad x^{\rm T}(k-1)\quad\cdots\quad x^{\rm T}(k-N)]^{\rm T}$, where $\Phi=\begin{bmatrix} A&0&\cdots&0&0\\ I&0&\cdots&0&0\\ 0&I&\cdots&0&0\\ \vdots&\ddots&\ddots&\ddots&\vdots\\ 0&0&\cdots&I&0\end{bmatrix},\Gamma=\begin{bmatrix}B\\0\\0\\ \vdots\\0\end{bmatrix},\Psi=\begin{bmatrix}I\\0\\0\\ \vdots\\0\end{bmatrix},\Xi=\begin{bmatrix}C&0&\cdots&0&0\\0&0&\cdots&0&C_\theta\end{bmatrix}.$ |
| Measurement augmentation | $N\times r_2+n$ | $Z(k)=[\ x^{\rm T}(k)\quad y^{*\rm T}(k)\quad\cdots\quad y^{*\rm T}(k-N)\ ]^{\rm T}$, where $\Phi=\begin{bmatrix} A&0&\cdots&0&0\\ C_\theta A&0&\cdots&0&0\\ 0&I&\cdots&0&0\\ \vdots&\ddots&\ddots&\ddots&\vdots\\ 0&0&\cdots&I&0\end{bmatrix},\Gamma=\begin{bmatrix}B\\C_\theta B\\0\\ \vdots\\0\end{bmatrix},\Psi=\begin{bmatrix}I\\0\\0\\ \vdots\\0\end{bmatrix},\Xi=\begin{bmatrix}C&0&\cdots&0&0\\0&0&\cdots&0&I\end{bmatrix}.$ |
| Sample-state augmentation | $2\times N$ | $Z(k)=[\ x^{\rm T}(k)\quad x^{\rm T}(k-N)\ ]^{\rm T}$, where $\Phi=\begin{bmatrix}A&0\\0&I\end{bmatrix},\Gamma=\begin{bmatrix}B\\0\end{bmatrix},\Psi=\begin{bmatrix}I\\0\end{bmatrix},\Xi=\begin{bmatrix}C&0\\0&C_\theta\end{bmatrix}.$ |

This method smooths the past $N$ states using the on-line measurements at the minor time instance. When the delayed off-line measurement is recorded, both off-line and on-line measurements are used to obtain smoothed estimates from $k-N$ to $k$.

## 3.2 State estimation in deterministic systems with delayed measurements

In this section, different techniques to incorporate non-uniform and delayed measurements in deterministic estimation strategies are presented. In these strategies, the system is considered as a hybrid system, i.e., the process is modeled as a continuous-time system and the effects of sensor and sampling are represented as a discrete-time system (see Fig. 6).

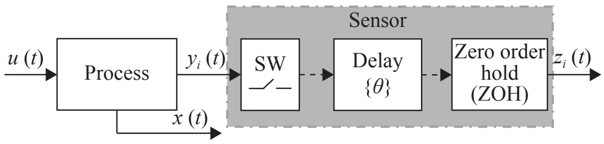


Fig. 6 Hybrid system scheme for deterministic estimates with delayed measurements. Source: modified from [13, 14]

According to Fig. 6, the process can be represented by

$$\begin{aligned} &\dot{x}\,(t) = f\,(x\,(t)\,, u\,(t)) \\ &x\,(0) = x_o \\ &y_i\,(t) = h\,(x\,(t)) \end{aligned} \tag{11}$$

where $y_i$ are the actual outputs, which are conditioned by the dynamics of each sensor. In this way, it is possible to apply systems with asynchronous sensors. The dynamics of the sensor is described by

$$z_i\,(t) = y_i\,(t-\theta)\,. \tag{12}$$

In the present review, such techniques have been grouped into four types of methods according to their structure: piece-wise observer[13], cascade or chain observer[14, 15], distributed observer[16], and partial state observer[19, 20]. These deterministic techniques solve the problems of state estimation independently or in stages. A brief description of the main characteristics of each method of this family follows.

### 3.2.1 Piece-wise observers

This type of observers is based on the theory of piece-wise continuous-time hybrid systems (PCHS), a particular class of hybrid systems characterized by autonomous switchings and controlled impulses[13]. In [13], a scheme composed of four linear piece-wise continuous-time hybrid systems (LPCHS), one reduced order discrete-time luenberger (RODL) observer, and one block of reconstruction calculation were proposed as shown in Fig. 7.

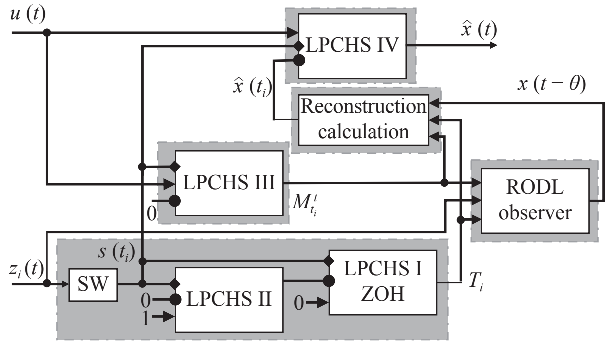


Fig. 7 Piece-wise continuous-time observer scheme. Source: modified from [13]

The inputs of the piece-wise continuous-time observer (PCO) are the delayed measurement and the current process input and output are the estimates of the current state $\hat{x}(t)$. The operation of the observer is defined in five stages described as follows:

1) First stage. Delayed measurement $z_i\,(t) = y_i\,(t-\theta)$ is switched to obtain a square wave signal $s(t_i)$. This signal is the input of the four LPCHS. Then, LPCHS I and LPCHS II are used to generate the delayed variable and the sample period of $T_i$[13].

2) Second stage. LPCHS III and input $u(t)$ are used to get $M_{t_{i-1}}^{t_i} = \int_{t_{i-1}}^{t_i} \mathrm{e}^{A(t_i-\theta)} Bu\,(\theta)\mathrm{d}\theta$.

3) Third stage. RODL observer is used to obtain the delayed estimated state $\hat{x}(t-\theta)$ according to the delayed measurement $y(t-\theta)$, $T_i$ and $M_{t_{i-1}}^{t_i}$.

4) Fourth stage. The reconstruction calculation block is used to get the discrete-time undelayed state $\hat{x}_i = A_d(T_i)\hat{x}_{i-1} + M_{t_{i-1}}^{t_i}$.

5) Fifth stage. Finally, using the LPCHS IV with the inputs of $u_s(t) = u(t)$ and $v_s(t) = \hat{x}_i$, it is possible to reconstruct the continuous-time undelayed state.

### 3.2.2 Chain or cascade observers

This method has two stages, one for state estimation and the other one to account for delay effects. Depending on the nature of the information, multi-sampling or with unknown delay, the stage of information processing may be previously or subsequently implemented at the estimation stage, as shown in Figs. 8 and 9. In this respect, since the estimation and data processing are performed by separated stages, it is possible to use either

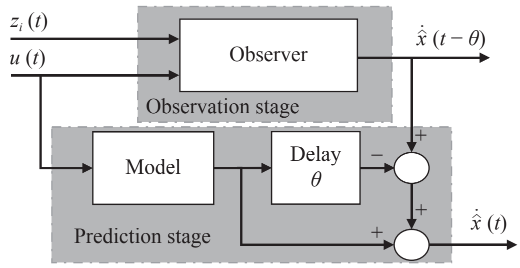


Fig. 8 Observer-predictor scheme for delay feed-back

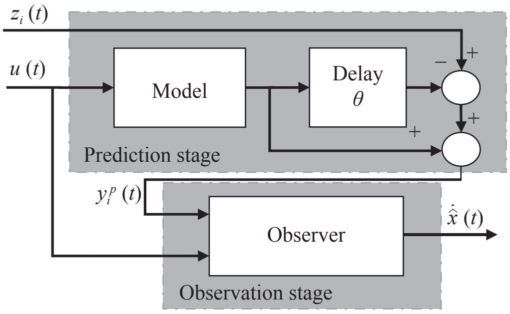


Fig. 9 Predictor-observer scheme for feed-forward delay

stochastic or deterministic state estimation techniques. However, in the literature, only deterministic estimation techniques are reported[14, 15, 22, 29].

Regarding the stage of information processing, different elements to reduce the effects of delay are used. In [29], a switching element is proposed to incorporate delayed measurements. Other authors propose to use a Smith predictor[30]. In the latter applications, the cascade observer-predictor is extended to use variable delay and multi-sampling, which are useful for applications with unknown delay. Due to the importance of cascade observer-predictor, in the present work, some features of this method are presented below.

The Cascade observer-predictor scheme (Fig. 8) has two stages. The first stage is called observation and normally is executed employing any observer structure. However, the problem considered in this review is to estimate the current state $x(t)$ when the measurements of the output are delayed, such that the output measurement at time $t$ is $y(t) = h(x(t-\theta))$ for some known constant delay $\theta \geq 0$. In this sense, a prediction stage is proposed to eliminate the delay effects on the measurement. The second stage is called prediction because a Smith predictor compensates the delay[15]. The Smith predictor is considered as

$$\dot{x}^p(t) = \dot{\hat{x}}^\theta(t) + f(x^p(t)) - f(x^p(t-\theta)) \qquad (13)$$

where the prediction of the current state is denoted by $x^p \in \mathbf{R}^n$ and $\hat{x}^\theta$ is the estimate $x$ subject to delayed output measurements (12). Moreover, with the system model (11) and the known delay $\theta$ for output measurements (12), it is possible to know the dynamics of the predicted states with and without delay $f(x^p(t))$ and $f(x^p(t-\theta))$, respectively.

The stability of the previously mentioned observer-predictor structure is such that the estimated state asymptotically/exponentially converges to the system trajectories (11) and (12), if the estimates provided by the observer converge in an asymptotic/exponential way to the delayed system state[15].

### 3.2.3 Distributed observers

A recent development in the field of state estimation for large-scale process systems is distributed observers[31]. This observer uses a network of interconnected estimators for each subsystem. This network consists of several estimator nodes. At each node, embedded computation, communication and power modules are included[16, 32]. A node acts as a local observer by computing estimates through its own model and available measurements. The communication module allows a sensor node to share information with other nodes in the network within a specified communication topology. This scheme is presented in Fig. 10.

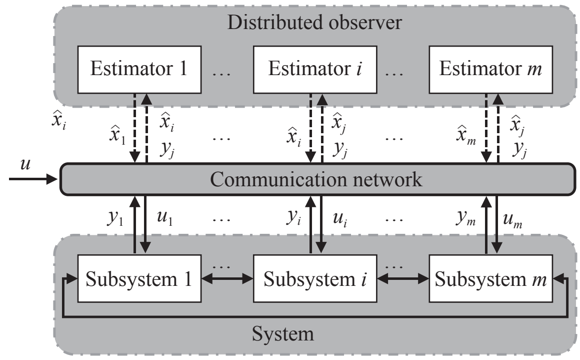


Fig. 10 Distributed observer scheme. Source: modified from [16]

In [16, 32], a distributed observer was proposed. There, for each node of the estimator, a predictor and a moving horizon state estimator (MHE) are embedded. In that proposed approach, the predictor subsystem handles communication delays and data losses directly while the local MHEs take advantage of the predictions given by the predictors. Applications of this distributed estimation scheme for systems with non-uniform and delayed information are focused on large-scale systems, where the system can be modeled as several sub-systems, such as electrical power systems[16, 32, 33].

### 3.2.4 Partial state observers

Finally, in this method, the estimation structures that use a reduced-order Luenberger observer with estimation or extrapolation approximations of delayed or asynchronous measurements are grouped. For example, in [17], a polynomial extrapolation is used. That is, slow measurements can be predicted for sample times where only fast measurements are available. Other applications are presented in [18–20] and will be discussed in Section 4.

## 3.3 Proposed taxonomy

This section shows a taxonomic proposal for available tools reported in the literature for incorporating non-uniform and delayed information in state estimation techniques. The proposed taxonomy has hierarchical levels as shown in Fig. 11.

In Fig. 11, the first hierarchical level is classified according to the type of estimator used. The second level classifies different methods to incorporate non-uniform

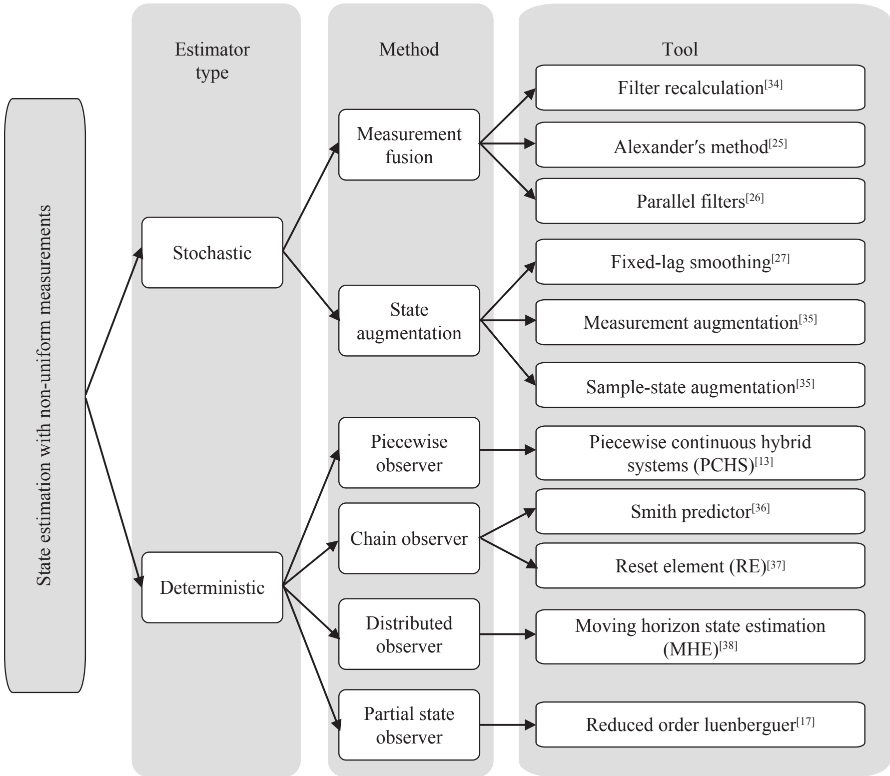


Fig. 11 Tools taxonomy for incorporating non-uniform and delayed measurements in state estimation techniques

and delayed information. Each method has different uses according to characteristics of information such as known delay, variable delay, multiple system outputs and multirate sampling, among others, as presented in Section 3. Finally, in the third level, some of the tools reported in the literature and their respective seminal references are presented to give the works supporting the proposal.

Table 2 presents a comparison of two of the most important structures reported in the literature for state estimation of systems in the presence of non-uniform and delayed measurements: the extended Kalman filter (EKF) with the state augmentation method and the chain observer-predictor algorithm. Table 2 shows some characteristics of the two families of state estimation approaches (stochastic and deterministic) for systems with non-uniform and delayed measurements. The analysis presented in Table 2 is supported by references[21, 22]. In these papers, estimation techniques were applied to estimating biomass in the $\delta$-endotoxins produced by *Bacillus thuringiensis* (*Bt*) where the measurements were non-uniform and delayed.

## 4 Applications of state estimators employing non-uniform and delayed information

In this section, some applications of different state estimators employing non-uniform and delayed information are presented in Table 3 as reported in the literature. The tools used in each application are analyzed according to the previously proposed taxonomy (see Fig. 11). Some applications for each of the six methods presented in the taxonomy are listed. In addition, for each application, some characteristics like author/year, estimator type, uses, system model, tools, and others are reported. The first column shows the reference. The second column is the estimator type, indicating the estimator structure. The next column shows the particular applications in which the proposed estimator was simulated and validated. The fourth column shows the type of required model for the proposed estimator structure. The fifth column describes the tool used to incorporate the non-uniform and delayed information. Finally, in the other characteristics column, some particular characteristics of the application are described. The applications are organized according to the method and chronological order of their publication.

In general, the applications in Table 3 show the following trends:

1) The asynchronous and delayed measurements are presented in various fields, from electromechanical systems to bioprocesses and navigation systems. However, there are still many real applications that could use this information, for example, large-scale systems. In addition, many of the published papers work with hypothetical models or laboratory-scale models, but there are not a lot of industrial applications.

2) There is a general trend to use stochastic estimat-

Table 2 Characterization of different approaches of state estimation for systems with non-uniform and delayed information

| Characteristic | KF with augmentation state | Chain observer-predictor |
|---|---|---|
| General information | For the incorporation of non-uniform measurements, in this estimation structure, the state space is increased according to the number of delayed samples of off-line measurement relative to the on-line measurement. | This structure is based in a cascade observer-predictor algorithm. The prediction stage offsets the effect of the delay in measurements. |
| Structure type | Compact and switched. It is necessary to commute the entire structure (model and estimator). | Cascaded in series and without feedback between stages. |
| Estimation technique allowed | The method for incorporating non-uniform and delayed measurements is designed to be used with the Kalman filter (KF) and its variants. | Different deterministic estimation techniques used with this structure are reported. Among them: Luenberguer observer (LO), sliding mode observer (SMO) and high gain observer (HGO). |
| Allowed information type | Delayed measurement. Multi-sampling of on-line and off-line measurements. The delay and sampling time of the off-line measurement can be variable. | System with delayed input or output. The delay can be variable. |
| Allowed model type | It can be applied to linear and non-linear systems. However, its use in non-linear systems is limited by the model linearization method. | The prediction stage accepts almost any type of model. Therefore, the type of model is limited by the observer used in the observation stage. |
| Observability test | The pair $(A, C)$ must be observable. That is, if observability of the system with on-line measurements is guaranteed, observability of the system with non-uniform measurements will be as well. | For linear time-invariant (LTI) systems, the delayed system is observable if the observability matrix has full rank. For time-varying systems and non-linear systems, tools are still limited. |
| Estimation error convergence | The estimation error convergence in each instance (major and minor) is guaranteed. However, the analysis of the commutation of each instance is a current research problem. | The structure of cascade in series allows the separate analysis of the convergence of each stage. In the prediction stage, convergence is guaranteed by the Smith predictor. In the observation stage, the convergence depends on the observer used. |
| Advantages | The required model transformation preserves the representation of the state space. This feature allows the extension of the method to other estimation techniques. | The structure of cascade in series allows the use of different deterministic estimation techniques. |
| Disadvantag-es | The transformation of the model is based on the linearized model at each sampling time. The size of the augmented state space is variable due to the delay variation. The covariance matrix $P$ must be returned for each commutation of the instances (major and minor). | It does not allow multi-sampling of the measurement. |

Table 3 Applications of state estimation for systems with nonuniform and delayed information

| References | Estimator type | Use | System model | Tools | Other characteristics |
|---|---|---|---|---|---|
| Stochastic methods based on measurement fusion | | | | | |
| [34] | Multi-rate extended Kalman filter | Applied to styrene polymerization process. | Non-linear chemical process | Parallel filters method for multi-rate extended Kalman filter (EFK) is used. | The proposed estimator is used for non-linear model predictive control. |
| [39] | Asynchronous Kalman filter | Application to a continuously stirred tank reactor (CSTR). | Non-linear system with asynchronous communication delay and different sampling rate | Filter recalculation method for handling asynchronous communication delay is used. Decentralized data fusion for handling different sampling rate is used. | For a CSTR, the following are simulated: an extended Kalman filter, an unscented Kalman filter and an asynchronous Kalman filter. The performance is evaluated with the root mean square error criteria. |
| [40] | Multi-rate moving horizon estimation | Two simulation examples. First, the estimation of molecular weight distributions in a styrene polymerization process. Second, simultaneous parameter and state estimation in a large scale binary distillation column. | Non-linear system with multi-rate measurements, unknown disturbances and measurement noise. | A variable structure of MHE is applied setting the MHE horizon large enough to cover both the sampling time and the arrival time of slow measurements. This proposal uses the filter recalculation method. | A strategy for updating the smoothed covariance matrix of the arrival cost based on non-linear programming (NLP) sensitivity is proposed. |

to be continued

Continued from Table 3

| References | Estimator type | Use | System model | Tools | Other characteristics |
|---|---|---|---|---|---|
| [41] | Intermediate step Kalman filter (ISKF) | Application for system navigation and tracking | Linear discrete-time system with asynchronous multi-rate sensor | State fusion based on parallel filters is the process to make combination of information coming from different sources of state estimates. | The estimate is computed recursively using a time-varying Kalman filter to handle missing measurements. The proposed estimator is compared with a conventional Kalman filter. |
| [42] | Modified Sigma-point Kalman filter | Numerical simulation example | A type of non-linear multirate multisensor asynchronous sampling dynamic system. | Parallel filters are used. The fused state estimate is generated using multiscale system theory and the modified Sigma-point Kalman filter. | This paper presents an extension to non-linear systems of their previous work[43]. |
| [9] | Kalman filter with data fusion | Two application cases are presented: a distillation column linear model (simulation) and the separation of bitumen from sand in oil. | Linear system with two types of measurements. The first is fast, regular, and provides delay-free measurements. The second one is infrequent and irregular with time-varying delays. | Data fusion method to incorporate delayed infrequent and irregular measurements. Performance indicators: average root mean square error (ARMSE) and average maximum computational time (AMCT). | Convergence and observability tests are not performed. |
| [44] | Moving horizon estimation delayed laboratory measurements | Application to a large-scale non-linear polymerization process. | Discrete-time model. Large-scale non-linear systems with multi-rate sampling measurements. | Filter recalculation method for handling multi-rate sampling measurements. Reference [40] results are extended. | It is investigated when laboratory measurements help to identify the plant model mismatch and to estimate convergence to the true state. |
| | | | Stochastic methods based on state augmentation | | |
| [8] | Variable dimension unscented Kalman filter (VD-UKF) | Application in simulation for state estimation in a bioreactor. | Non-linear system state estimation incorporating infrequent, delayed and integral measurements. | Sample-state augmentation method. Stability conditions for VD-UKF are proposed. The covariance matrix of a priori estimation error is related to the observability of the system. | A concept that allows greater weight to laboratory measurements, called integral measurements, is included. The assumption of invertibility of Jacobian matrices in the existing methods is no longer valid for VD-UKF. Therefore, a relaxation of the stability condition for VD-UKF is proposed. |
| [45] | Multirate Kalman filter | Applied to a numerical case | Non-linear system with multi-rate measurements | Sample-state augmentation method to incorporate multi-rate measurements. | Low frequency measurements are incorporated in the estimation as proposed to calibrate the Kalman filter, but a mismatched filter to model is included. |
| [46] | Kalman filter with single/multiple measurement packets | Applied to a linear numerical case | Discrete-time LTI system | A modification of the sample-state augmentation method. The state is updated only at each sampling processing instant of the major instance and all previous estimates (of the minor instance) are left unchanged. | A non-probabilistic approach by time-stamping the measurement packets is proposed. |
| [11] | Kalman filter with irregular sampling and time-varying delays | Applied to a linear numerical case | Discrete-time LTI system | Sample-state augmentation method is used. This method is similar to that proposed in [46], but it is modified to reduce the computational cost of the algorithm. | The proposal to incorporate asynchronous measurements is compared by simulation with the augmented state method[7] and the algorithm in [46]. |
| [23] | Cubature Kalman filter with delayed measurements | Applied to the state estimation for penicillin and industrial yeast fermentation processes | Non-linear discrete-time system | Sample-state augmentation method to incorporate multi-rate measurements. | The results show the application of the estimator with real data. |

to be continued

Continued from Table 3

| References | Estimator type | Use | System model | Tools | Other characteristics |
|---|---|---|---|---|---|
| [47] | Finite-dimensional sub-optimal filter | A new Kalman-Bucy filter | LTI continuous-time systems with known and bounded measurement delays. | Infinite dimensional state space representation is used to manage delay systems, this method is based on augmented state. | The proposed approach allows a precise characterization of the relationship between measurement delay and estimation error covariance. |
| [21] | Extended Kalman filter with delayed measurements | Applied to biomass concentration estimation in the $\delta$-endotoxins production of *Bacillus thurringiensis*. | Non-linear systems with delayed and multi-rate measurements | Fixed lag smoothing method to incorporate delayed measurements. | A methodology for incorporating delayed and multisampling measurements in non-linear state estimation techniques is proposed based on [7]. |
| Deterministic methods based on piece-wise observers | | | | | |
| [13] | Piece-wise continuous observer (PCO) | Electromechanical processes. Real application for a conveyor belt with artificial vision system. | Linear system with variable sampling time and delayed measurements. | The process is modeled as a piece-wise continuous-time hybrid system and reduced-order discrete-time Luenberger observer is used to obtain the delayed estimated state. | Assumes that the linear system is observable. |
| [48] | Two types: Luenberger-type observer and observer based on particle filtering algorithms. | Application in simulation for two generic bi-modal piece-wise affine systems. | Discrete-time LTI system with continuous and discrete signals (hybrid). | In the deterministic case: Lyapunov functions, Luenberger observers, linear matrix inequalities; In the discrete case: particle filtering, Monte Carlo methods. | In deterministic case: globally asymptotically stable error, computationally easy to implement. In stochastic case: stability if the number of particles goes to infinity; however more stability tests are required. |
| Deterministic methods based on chain observers | | | | | |
| [49] | Chain observer for delayed non-linear systems | Numerical example for a continuous-time non-linear system | Continuous-time non-linear systems with delay | The proposed algorithm is composed of $m$ observers in a chained form, each one estimating the state at a given fraction of the output delay. The last observer estimates the current state. | Exponential convergence of the estimate is ensured if the integer $m$ is sufficiently large. |
| [50] | Observer for discrete-time Lipschitz non-linear systems with delayed output | Numerical example for a discrete-time non-linear system | Discrete-time Lipschitz non-linear systems with delayed output | This observer consists of a chain structure that estimates the system state at different delayed time instants. Using the discrete Gronwall inequality, the exponential stability of the estimation error is guaranteed. | The proposed observer is an extension of [49] in discrete-time systems. |
| [29] | Reset observers | Numerical example | Linear time-varying delay systems | The reset element is used in a Luenberger observer for a linear system to incorporate delayed measurements. | It is assumed that the system is observable. Stability conditions are developed based on the Lyapunov-Krasovskii function and linear matrix inequalities (LMIs). |
| [51] | Chain observer for MIMO non-linear systems | Applied to a problem of hyperchaos synchronization when the measurements are stored in data packets before the arrival to the processing unit *Bacillus thurringiensis*. | Multi-input and multiple-output non-linear systems with time-varying measurement delays | The Lyapunov-Razumikhin approach was used to prove the asymptotical convergence to zero of the observation error. | This observer has two characteristics: a single step and uniform structure. These features allow to treat vector measurements (MIMO) and achieve the decomposition of the prescribed exponential error, and to deal with the case where the prescribed exponential convergence can not be reached by a single-pass observer. |

to be continued

ors when considering real plants. In this case, models are assumed discrete and linear time invariant, probably be-

Continued from Table 3

| References | Estimator type | Use | System model | Tools | Other characteristics |
|---|---|---|---|---|---|
| [14] | Cascade combination of an output predictor and an attitude observer or filter | Attitude estimation problem when sampled and delayed vector measurements are available. | The observer is proposed for the exact continuous-time non-linear attitude kinematics model with 3-dimensional rotation, often donated SO3. In addition, the sampling and delays in the sensor are modeled as a zero order hold (ZOH). | Cascade combination of a predictor with an attitude observer or filter in which the predictor compensates for the effect of sampling as well as delays in vector measurements and the filter or observer processes the predicted outputs and estimates the attitude. | The predictor compensates for the effect of sampling and delays in vector measurements and provides continuous-time predictions of outputs. These predictions are then used in an observer or filter to estimate the current attitude. |
| [15] | State estimation for non-linear systems with delayed output measurements. | Numerical example for continuous-time non-linear system. | Continuous-time non-linear system with delayed measurements. | Rigorous stability analysis for globally Lipschitz systems to demonstrate the current estimation state convergence (asymptotically/exponentially) to the delayed system state. | The observer takes the delayed outputs and estimates the delayed states of the system. Then a predictor takes the delayed estimates from the observer and fuses them with the current input measurements of the system to compensate for the delay. |
| [22] | Observer plus predictor in cascade. | Applied to biomass concentration estimation in the $\delta$-endotoxins production of *Bacillus thurringiensis*. | Non-linear systems with delayed measurements. | Chain observer composed of two stages. First, an estimation stage based in a class of second order sliding mode algorithms and then a prediction stage that compensates for the delay effect on the measurements. | Convergence proof and numerical simulations showed the feasibility of the proposed cascade observer-predictor. |
| Deterministic methods based on distributed observers | | | | | |
| [33] | Reduced-order distributed functional observers | Distributed functional observers design technique for a class of interconnected systems with delays. Applied to a numerical example. | Interconnected linear systems with the presence of time delays in the interconnections. | The fundamental concept of the observer is that by having measurement information transferred from other subsystems, the highly restricted existence conditions as in the case of a totally decentralized observer implying an unknown input observer (UIO) design scheme can be relaxed. | The scheme does not require the exchange of information among the local observers. |
| [32] | An observer-enhanced moving horizon state estimator (MHE) | A class of non-linear systems composed of several interconnected subsystems and subject to time-varying communication delays. | Application in simulation to the reactor-separator process composed of two connected continuous-time stirred tank reactors (CSTR) and one flash tank separator. | In each node of the distributed estimator, a predictor and a moving horizon state estimator are embedded. In the proposed approach, the predictor's subsystem handles communication delays and data losses directly while the local MHEs take advantage of the predictions given by the predictors. | Applications of this distributed estimation scheme for systems with non-uniform and delayed information are focused on large-scale systems. |
| [16] | Distributed moving horizon estimator (DMHE) | Chemical processes. Real application for a sequence of two reactors and one spray tower. | Non-linear system subject to data loss and communication delays. | Distributed structure formed by a deterministic auxiliary observer and local MHEs. The auxiliary observer determines the trust region of the actual states and local MHEs optimize the estimated states within the region. Data loss is determined with a permissible constraint time. | Stability is guaranteed with the Lyapunov-based model predictive control[52] |

to be continued

cause most of the tools have been developed for this type of models. However, linear models do not always repres-

Continued from Table 3

| References | Estimator type | Use | System model | Tools | Other characteristics |
|---|---|---|---|---|---|
| Deterministic methods based on partial state observers | | | | | |
| [17] | Multirate reduced-order Luenberger state observer | Non-linear chemical process with multirate measurements. Application to a polymerization reactor. | Continuous-time reduced order non-linear observers are used for state estimation in polymerization reactors. | Using polynomial extrapolation, the slow measurements can be predicted for sample times where only fast measurements are available. | The authors also describe a method for tuning the observer gains ensuring that the estimation error asymptotically decays to zero. |
| [18] | Non-linear observer design in the presence of delayed output measurements | Biological reactor example. | Designing a state-dependent gain for a particular class of non-linear systems. | Non-linear observer with a state-dependent gain which is computed from the solution of a system of first-order singular PDEs. | Convergence of the estimation error to zero is defined under a set of conditions. |
| [19] | Observer with interval time-varying delay | Numerical example for a continuous-time non-linear system | Non-linear systems with time-delay and uncertain non-linearity. Luenberger observer. | Using the mean-value theorem and constructing the Lyapunov-Krasovskii functional, the convergence conditions for the non-linear observer are established. | The process model must be adjusted to a predefined structure, in which the delay is included in a part of the dynamics of the state space. |
| [20] | Partial state estimation for linear systems with output and input time delays | Numerical examples for linear systems with output delay and instantaneous input. | Linear continuous-time systems that are subject to different time delays in both the measured output and control input. | The proposed observer estimates system state functionals in the case of different time delays present in both the output and input of the system. | To guarantee system stability, a proposal to augment a quadratic term of the chosen Lyapunov-Krasovskii functional with three delay range-decomposed integral terms is presented. |

ent the actual behavior of the processes, since they can be removed from the linearization conditions or undergo significant changes in their inputs. Therefore, developments that focus on non-linear models are required.

3) Regarding the stochastic tools, there is a very marked use of Kalman filter and its modifications. That is, there is a lot of credibility in this algorithm due to its ability to handle noise and modeling uncertainty. But Kalman filtering assumes a model with linear dynamics and it supposes noise with a Gaussian probability distribution function. Additionally, the Kalman filter algorithm needs initial values for the estimate state and for the covariance matrix, but such values are not easy to find, and therefore they are assumed by the designer.

4) The methods based on "state augmentation" present more recent applications than the methods based on "fusion measurement". In this regard, some authors defend the greater favorability for the use of methods based on "state augmentation", due to its possible extension to deterministic estimation techniques[7]. However, these methods present similar problems to those described in the previous item with the use of Kalman filter. Moreover, the extension of these methods to deterministic estimation techniques is not a trivial matter because the strong switching required at each measurement instance (major and minor) causes problems with the convergence of the estimator.

5) Regarding the deterministic estimators, there is a strong tendency to use them to validate the results in numerical cases. These estimators use non-linear models. Most of the deterministic structures use the observer-predictor combination to produce a well-performing estimate. However, there is a lack of application of these methods in real cases that consider not only the management of measurement and delayed input but also the multi-sampling. In addition, a greater diffusion of the results is necessary.

6) On the other hand, the deterministic tools for the incorporation of non-uniform and delayed information basically consist of distributed observers due to the great influence that this area has recently had in the control field. These tools, in many cases, designed in blocks or with hybrid structures, allow a greater flexibility of expansion of the system to be observed and its application in large scale systems. But in turn, this flexibility causes problems of standardization and generalization of the tools or methods, so they are developed for specific problems.

7) To achieve the technological transfer of state estimators using non-uniform and delayed information, it is necessary to validate the methods in different real applications. For example, in large industries such as oil, sugar or textile, it would be useful to promote the use of all information obtained by sensors and results of laboratory tests for estimation and control of these processes. It is even possible to extend the use of non-uniformed and delayed information in large-scale systems such as hydro-meteorological networks, interconnected electrical systems, water systems or the integrated mass transport system.

8) Some recent works, which were not included in Table 3 because they were applied to control systems and identification, propose novel techniques and tools for systems with non-uniform and delayed information. These

novel techniques and tools could be extrapolated to the problem of state estimation. For example in [53], a recursive Bayesian identification algorithm with covariance resetting is proposed to identify systems with non-uniformly sampled input data. In [54], a robust finite-time $H_\infty$ control for delayed time-varying system is proposed. Based on these works, tools such as: recursive least squares, receding horizon estimation, $H_\infty$ control and linear matrix inequalities could be used as a new proposal for the estimation of state in the presence of non-uniform measurements.

## 5 Conclusions

In this paper, a set of useful definitions acting as a framework for research in state estimation with non-uniform and delayed information were reviewed and information sources found from industrial processes were characterized. Also, different methods to incorporate non-uniform and delayed measurements in state estimation techniques were presented in general terms. After that, a taxonomy to collect and classify different methods and tools reported for estimation was proposed. This classification was performed according to the type of estimator and model used in each technique. In addition, the taxonomic classification considers the phenomenon of acquisition, storage and use of non-uniform and delayed information from real applications and also incorporates both stochastic and deterministic estimation techniques. Finally, through proposed taxonomy, different reported applications in the literature were summarized in a table and then analyzed and criticized. A critical analysis of the references showed that it is still necessary to investigate modeling uncertainty, to reformulate the state augmentation method for incorporating non uniform and delayed information in deterministic estimators, to expand deterministic methods to other phenomena of sensor networks such as multi-sampling, to improve the performance of these types of estimators in the case of variable parameters, and finally, to apply the techniques developed in real processes, particularly in large-scale systems.

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