International Journal of Automation and Computing 13(6), December 2016, 615-623
DOI: 10.1007/s11633-016-1013-3

Design of Decentralized Multi-input Multi-output Repetitive Control Systems

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Abstract: This paper presents the design of decentralized repetitive control (RC) for multi-input multi-output (MIMO) systems. An optimization method is used to obtain a RC compensator that ensures system stability and good tracking performance. The designed compensator is in the form of a stable, low order, and causal filter, in which the compensator can be implemented separately without being merged with the RC internal model. This will reduce complexity in the implementation. Simulation results and comparison study are given to demonstrate the effectiveness of the proposed design. The novelty of design is also verified in experiments on a 2 degrees of freedom (DOF) robot.

Keywords: Repetitive control (RC), compensator, multi-input multi-output (MIMO), decentralized, optimization, two degrees of freedom robot.

1 Introduction

Tracking periodic commands are common tasks found in many control systems such as disk drive, optical disc player, and pick-and-place robot. Repetitive control (RC) gives superior performance for tracking references compared to a non-predictive control schemes such as proportion integral (PI) and proportion integration differentiation (PID) controller[1]. This is due to the capability of RC to learn the periodic signal values, and then generate them as an output. RC consists of two parts; an internal model and a RC compensator. The internal model refers to the internal model principle (IMP) originated from Francis and Wonham[2], which states that the reference model needs to be attached to the feedback loop in order to achieve zero tracking error. The compensator is part of RC used to ensure closed-loop system stability.

RC compensator designs for single-input single-output (SISO) systems have been widely investigated in $[3-13]$. A compensator in the form of inverse of the plant model was proposed in $[3-6]$, where the design aims to perfectly cancel the phase of the plant model. In [7, 8], a compensator is designed based on pole placement, and it is obtained by solving diophantine equation. Panomruttanarug and Longman[9] uses optimization to obtain a compensator that approximates the inverse of the plant model. In [10] , a compensator in the form of non-causal FIR filter was designed based on Taylor expansion. A compensator design based on direct adaptive control to handle unknown and time-varying plant model was proposed in [11]. Design of a discrete output-feedback compensator for a class of linear plants with periodic uncertainties was investigated in [12]. In [13], a compensator in the form of causal IIR filter is proposed, where the compensator is obtained by solving the minimization problem.

Unlike for SISO RC systems, there are still few RC designs for multi-input multi-output (MIMO) systems, where some of them were found in [14–19].

In [14], a new MIMO RC structure was proposed for periodic wind disturbance rejection in fixed-speed wind turbines and variable-speed wind turbines. While, a new compensator structure which consists of adaptive internal model for minimum and non-minimum phase linear MIMO systems was given in [15]. An MIMO RC structure for tracking periodic references for uncertain linear systems subject to control saturation was proposed by Flores et al.[16]. Wang et al.[17] proposed an MIMO RC structure for tracking periodic signals by using receding horizon control with frequency decomposition of the reference signals. Jeong and Faben[18] proposed a discrete-time RC compensator called as phase cancellation inverse (PCI) matrix. The proposed compensator operates by compensating the phase lag in the diagonal elements of the plant model. The idea is initiated from the zero phase tracking error controller (ZPETC)[5] design for SISO system, which aims to cancel the phase response of the plant. Xu[19] proposed an optimization based RC compensator to mimic the inverse of the MIMO model. The approaches[18, 19] are based on the centralized MIMO design which results in a compensator with the same dimension as the plant. This implies that if we have an $m \times m$ MIMO system (m^2 transfer functions), then we need to have m^2 RC sub compensators. Moreover, the designs also end up with a non-causal compensator that needs to be merged with the internal model to make it realizable. This raises

Research Article
Manuscript received February 22, 2015, accepted November 16, 2015; published online July 25, 2016
Recommended by Associate Editor Min Wu

the complexity of the implementation, especially when the order of the internal model is high.

The fact that most MIMO control problems including majority of industrial process applications are still based on decentralized controller[20, 21] gives a motivation to design an RC compensator on decentralized basis. A decentralized control also exhibits several advantages such as flexibility in operation, simplified design and tuning, etc[22]. In spite of the above practical benefits, the decentralized control cannot surpass the performance of centralized control in the presence of strong couplings[21]. However, this drawback of decentralized control has been extensively dealt in many literatures[21−24].

This paper presents an RC compensator design of MIMO systems based on the decentralized control. Early efforts for such control designs have been made in [25, 26]. Using decentralized control, the proposed design will only have m sub compensators, which is far less than that in [18, 19]. Another advantage of the proposed design is that the sub compensators are in a low order, stable and causal form. This form of compensator can be implemented independently without being merged to the internal model that is normally in high order. This feature can reduce the complexity in the design implementation. A complete series of simulation, comparison study and experiments is carried out to demonstrate the effectiveness of the proposed algorithm.

This paper is organized as follows. Section 2 presents an overview of MIMO RC system, which covers the general structure of MIMO RC feedback control and its stability analysis. Section 3 describes a design method to obtain the proposed compensator. Simulation results and a comparison study are provided in Section 4. Experimental results of the 2 degrees of freedom (DOF) robot plant are presented in Section 5. Section 6 concludes the paper.

2 MIMO RC System

The general structure of MIMO RC system is shown in Fig. 1, where $\boldsymbol{G}(z)$ is the plant model with $m \times m$ transfer functions, $\boldsymbol{G}_{rc}(z)$ is the repetitive controller, $\boldsymbol{U}_{rc}(k)$ is the repetitive control signal, $\boldsymbol{Y}(k)$ is the tracking output, $\boldsymbol{E}(k)$ is the tracking error, $\boldsymbol{R}(k)$ is reference signal, and $\{\boldsymbol{U}_{rc}(k), \boldsymbol{Y}(k), \boldsymbol{E}(k), \boldsymbol{R}(k) \in \boldsymbol{R}e^m\}$ [1].

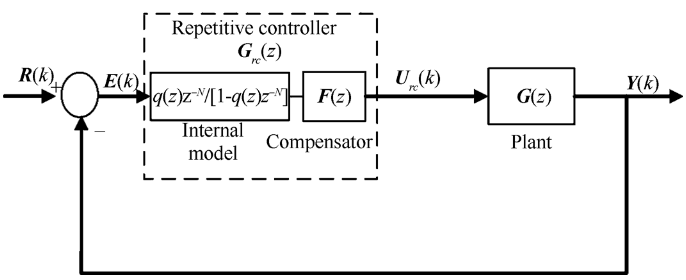

Fig. 1 General structure of MIMO RC system

[1]Note: Matrix notation is in bold style

The transfer function of discrete-time MIMO RC is

$$\boldsymbol{G}_{rc}(z) = \boldsymbol{F}(z)\frac{q(z)z^{-N}}{1-q(z)z^{-N}} \tag{1}$$

where $N = \frac{T_s}{T}$, N is number of samples per reference period, T_r is the reference period, T is the sampling period, $q(z)$ is a zero-phase low pass filter, and $\boldsymbol{F}(z)$ is a compensator matrix.

The term $\left[\frac{q(z)z^{-N}}{(1-q(z)z^{-N})}\right]$ in (1) is sometimes called a modified internal model, and behaves as a generator of periodic signal. The use of zero-phase low pass filter $q(z)$ is similar to that in SISO case, which aims to improve robustness against unmodeled dynamics[18, 27]. The compensator matrix $\boldsymbol{F}(z)$ here is part of RC required to stabilize closed-loop of the MIMO RC system. $\boldsymbol{F}(z)$ generally has the same dimension as the plant model, and it is represented in following matrix:

$$\boldsymbol{F}(z) = \begin{bmatrix} f_{11}(z) & \cdots & f_{1m}(z) \\ \cdots & f_{22}(z) & \cdots \\ f_{m1}(z) & \cdots & f_{mm}(z) \end{bmatrix}. \tag{2}$$

To assess the stability of MIMO RC system, we need to derive the characteristic equation of the closed-loop system. The transfer function from $\boldsymbol{R}(z)$ to $\boldsymbol{Y}(z)$, and the tracking error $\boldsymbol{E}(z)$ of the MIMO RC system shown in Fig. 1 are given as follow:

$$\frac{\boldsymbol{Y}(z)}{\boldsymbol{R}(z)} = \frac{q(z)\boldsymbol{F}(z)\boldsymbol{G}(z)}{(z^N\boldsymbol{I} - (\boldsymbol{I} - \boldsymbol{F}(z)\boldsymbol{G}(z))q(z))} \tag{3}$$

$$\boldsymbol{E}(z) = \frac{z^N - q(z)}{(z^N\boldsymbol{I} - (\boldsymbol{I} - \boldsymbol{F}(z)\boldsymbol{G}(z))q(z))}\boldsymbol{R}(z). \tag{4}$$

Let $\boldsymbol{S}(z)$ be

$$\boldsymbol{S}(z) = z^N\boldsymbol{I} - (\boldsymbol{I} - \boldsymbol{F}(z)\boldsymbol{G}(z))q(z) \tag{5}$$

where I is an $m \times m$ identity matrix.

The stability of the MIMO RC system above can be examined from the location of the zeros of the characteristic equation. The characteristic equation can be obtained by calculating the determinant of the transfer function $\boldsymbol{S}(z)$ ($\det \boldsymbol{S}(z)$). For a stable system, all zeros of $\det \boldsymbol{S}(z)$ have to be inside the unit circle. Calculating $\det \boldsymbol{S}(z)$ can be troublesome due to the order of N is normally large. For instance, a robot performing a repetitive task with a period of 1 s at sampling period of 1 ms, we have $N = 1\,000$[28]. Therefore, examining the location of zeros of $\det \boldsymbol{S}(z)$ is less effective. Instead of examining the zeros of the characteristic equation, the stability can be assessed in simpler way as shown in (6). The overall MIMO RC system is stable if the following conditions are satisfied[19]:

1) $\boldsymbol{G}(z)$ is a stable MIMO plant.

$$2)\ \det(\boldsymbol{I} - \boldsymbol{F}(z)\boldsymbol{G}(z))q(z)) < 1,\ \forall\, 0 < \omega < \frac{\pi}{T}. \tag{6}$$

A stable MIMO plant is the first requirement in the design of RC, while the stability condition (6) becomes a basis in the design of $\boldsymbol{F}(z)$ since $\boldsymbol{G}(z)$ and $q(z)$ are considered as known variables. The stability condition (6) also ensures that the tracking error of RC MIMO system converges to zero for all possible N, and for all possible initial conditions, if and only if the magnitude of $\det((\boldsymbol{I}-\boldsymbol{F}(z)\boldsymbol{G}(z))q(z)))$ is less than one for all frequencies up to the Nyquist frequency[19].

Thus, the compensator matrix $\boldsymbol{F}(z)$ needs to be carefully designed in order to ensure the stability of the MIMO RC system. Later on, the stability condition (6) is used as a constraint in the proposed optimization problem to obtain $\boldsymbol{F}(z)$.

3 RC compensator design

Let the plant be an MIMO system with square matrix as shown in the equation below:

$$\boldsymbol{G}(s)=\begin{bmatrix} g_{11}(s) & \cdots & g_{1m} \\ \cdots & g_{22}(s) & \cdots \\ g_{m1}(s) & \cdots & g_{mm}(s) \end{bmatrix}. \tag{7}$$

The plant $\boldsymbol{G}(s)$ has an equal number of outputs and inputs, where the relations between outputs and inputs are given as follows:

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ \cdots \\ y_m(t) \end{bmatrix} = \begin{bmatrix} g_{11}(s)u_1(t)+\cdots+g_{1m}(s)u_m(t) \\ g_{21}(s)u_1(t)+\cdots+g_{2m}(s)u_m(t) \\ \cdots \\ g_{m1}(s)u_1(t)+\cdots+g_{mm}(s)u_m(t) \end{bmatrix} \tag{8}$$

where $y_1(t),\cdots,y_m(t)$ are the 1st to m-th plant outputs, and $u_1(t),\cdots,u_m(t)$ are the 1st to m-th plant inputs.

We use an assumption that the MIMO system can be treated on decentralized basis as motivated in [20, 21]. Decentralized basis aims to approximate the MIMO system by a set of SISO systems. The decentralized term here means ignoring dynamics that result in weak interactions. Each of the system outputs is approximated from the input response that gives dominant contribution. Therefore, the degree of interaction is necessary to be quantified. Relative gain array (RGA) introduced by [29] is one of the techniques termed as dominant interaction control method that can be used to determine the best input output pairings for multivariable control.

RGA is defined as matrix Λ which is formulated as follows:

$$\Lambda=\boldsymbol{G}(0).*[\boldsymbol{G}^{-1}(0)]^{\mathrm{T}} \tag{9}$$

where $\boldsymbol{G}(0)$ and $\boldsymbol{G}^{-1}(0)$ are system dc gain matrix and its inverse, notations $.*$ and T operate as element wise multiplication and transpose of matrix respectively.

In particular, the best pairings are picked up from the entries of Λ that are large. Suppose the diagonal entries of Λ have larger values than the off-diagonal entries, then the best pairings are $[y_i,u_j]_{(i,j)=(1,1),(2,2),\cdots,(m,m)}$. This also means that we consider the transfer functions $[g_{ij}(s)]_{(i,j)=(1,1),(2,2),\cdots(m,m)}$ as the strong dynamics that give dominant contribution to the plant outputs. In contrary, if the off-diagonal entries have largest values, then the best pairings are $[y_i,u_j]_{(i,j)=(1,m),(2,m-1),\cdots,(m,1)}$ pairings.

To design compensator, we firstly need the information regarding the best pairings obtained from the RGA analysis. For instance, RGA analysis suggests $[y_i,u_j]_{(i,j)=(1,1),(2,2),\cdots,(m,m)}$ pairings, then $\boldsymbol{F}(z)$ will only have elements in the diagonal:

$$\boldsymbol{F}(z)=\begin{bmatrix} f_{11}(z) & 0 & 0 \\ 0 & f_{22}(z) & 0 \\ 0 & 0 & f_{mm}(z) \end{bmatrix} \tag{10}$$

where $f_{11}(z),\cdots,f_{mm}(z)$ are m- sub compensators to be designed.

$\boldsymbol{F}(z)$ above consists of m sub compensators, which are far less compared to the general compensator shown in (2). From (10), the sub compensator[2] $f_{ji}(z)_{(j,i)=(1,1),(2,2),\cdots,(m,m)}$ will be designed to compensate the dynamics of $g_{ij}(z)_{(i,j)=(1,1),(2,2),\cdots,(m,m)}$, where $g_{ij}(z)$ is a discrete model of $g_{ij}(s)$ at sampling period T. Thus, there are m sub compensators needed to be designed. Adopting the design method in [13], the sub compensator here is designed to minimize the magnitude response of the following stability condition:

$$|(1-f(z)g(z))q(z)|<1,\ \forall\, 0<\omega<\frac{\pi}{T} \tag{11}$$

where $f(z)$ and $g(z)$ are the compensator and plant model of SISO RC system, respectively.

Stability condition (11) can be interpreted that an ideal compensation of the plant $g(z)$ can be achieved if the magnitude response of $((1-f(z)g(z))q(z))$ is zero for all frequencies up to the Nyquist. This ideal compensation can be obtained if we choose the compensator $f(z)$ as the exact inverse of the plant model. However, the inverse of the plant model is sometimes not available due to uncertainties[30], and the inverse model is almost unstable due to zeros of discrete-time plant are close to the unit circle[31]. Moreover, the inverse of discrete-time plant model will always be in non-causal form. Instead of obtaining an exact inverse of the plant $g(z)$, $f(z)$ can be designed in the form of any filter as long as it gives a small value to left hand side of (11) for all frequencies up to Nyquist. Here, optimization method is employed to find $f(z)$ that minimizes that term.

To simplify the notation, let us rewrite $f_{ji}(z)$ as $f_j(z)$, where $j=1,2,\cdots,m$ corresponds to $(j,i)=(1,1),(2,2),\cdots,(m,m)$ respectively. The sub compensator $f_j(z)$ has the following stable causal form.

$$f_j(z)=\frac{c_{j0}z^{n_j}+c_{j1}z^{n_j-1}+\cdots+c_{jn_j}}{(z-p_{j1})(z-p_{j2})\cdots(z-p_{jn_j})},\ n_j>0 \tag{12}$$

where $c_{j0},c_{j1},\cdots,c_{jn_j},p_{j1},p_{j2},\cdots,p_{jn_j}$ are parameters of $f_j(z)$ to be obtained.

[2]Note: the sub compensator notation is in $f_{ji}(z)$ instead of $f_{ij}(z)$. This is due to the output of $\boldsymbol{F}(z)$ is the input of $\boldsymbol{G}(z)$.

Let the following objective function is defined for a single sub compensator $f_j(z)$:

$$h_j = \sum_{k=1}^{\frac{N}{2}} |(1 - f_j(z)g_j(z))q(z)|_{\omega=\omega_k}, \forall\omega_k = \frac{2\pi k}{NT} \qquad (13)$$

where ω_k is k-th harmonic of the reference signal (rad).

Since there are m sub compensators, the total objective function is given as

$$h_{Total} = \sum_{j=1}^{m} h_j. \qquad (14)$$

Now, we propose the optimization problem as shown in (15). The first condition in (15) consists of $\sum_{j=1}^{m} n_j$ bound constraints which guarantee that all poles of $f_j(z)_{j=1,2,\cdots,m}$ are inside the unit circle. The positive constant γ ensures that the obtained poles are within a safe distance from the unit circle.

$$\min_{[c_{10},\cdots,c_{1n_1},\cdots,c_{m0},\cdots,c_{mn_m},p_{11},\cdots,p_{1n_1},\cdots,p_{m1},\cdots,p_{mn_m}]} h_{Total}$$

Subject to:

$$1)\ \begin{bmatrix} -1+\gamma \\ \vdots \\ -1+\gamma \\ \vdots \\ -1+\gamma \\ \vdots \\ -1+\gamma \end{bmatrix} < \begin{bmatrix} p_{11} \\ \vdots \\ p_{1n_1} \\ \vdots \\ p_{m1} \\ \vdots \\ p_{mn_m} \end{bmatrix} < \begin{bmatrix} 1-\gamma \\ \vdots \\ 1-\gamma \\ \vdots \\ 1-\gamma \\ \vdots \\ 1-\gamma \end{bmatrix}$$

$$2)\ |\det(\boldsymbol{I} - \boldsymbol{F}(z)\boldsymbol{G}(z))q(z))|_{\omega_k} < 1 - \epsilon, \forall\omega_k = \frac{2\pi k}{NT}, \quad k = 1, \cdots, \frac{N}{2} \qquad (15)$$

where $p_{11}, \cdots, p_{1n_1}$ are n_1 poles of $f_1(z)$, $p_{m1}, \cdots, p_{mn_m}$ are n_m poles of $f_m(z)$, $|\det(\boldsymbol{I} - \boldsymbol{F}(z)\boldsymbol{G}(z))q(z))|_{\omega_k}$ is the magnitude of $\det[(\boldsymbol{I} - \boldsymbol{F}(z)\boldsymbol{G}(z))q(z)]$ at frequency ω_k, and γ and ϵ are small positive constants.

The second condition guarantees that the MIMO RC system is stable within a positive margin of ϵ. Solving above minimization problem will give all parameters of the sub compensators ($\sum_{j=1}^{m} 2n_j + 1$ parameters). Thus, the compensator matrix $\boldsymbol{F}(z)$ can be obtained.

The design procedure to obtain the compensator matrix $\boldsymbol{F}(z)$ can be summarized as follows:

1) Perform RGA analysis (9) to obtain the best pairings.

2) Solve the optimization problem (15) by firstly choosing the following variables; a filter $q(z)$, compensator order $[n_j]_{j=1,\cdots,m}$, positive constants γ and ϵ.

Remark 1. In case that the off-diagonal entries of Λ have largest values, then $\boldsymbol{F}(z)$ will have elements in the off-diagonal. Here, the sub compensators $f_{ji}(z)_{(j,i)=(1,m),(2,m-1),\cdots,(m,1)}$ will be obtained, where $f_{1m}(z), f_{2(m-1)}(z), \cdots, f_{m1}(z)$ are designed to compensate the dynamics of $g_{m1}(z), g_{(m-1)2}(z), \cdots, g_{1m}(z)$ respectively. Let us rewrite $f_{ji}(z)$ as $f_j(z)$, where $j = 1, 2, \cdots, m$ corresponds to $(j, i) = (1, m), (2, m-1), \cdots, (m, 1)$. Let us also rewrite $g_{ij}(z)$ as $g_i(z)$ where $i = 1, 2, \cdots, m$ corresponds to $(i, j) = (1, m), (2, m-1), \cdots, (m, 1)$. In this notation, $f_1(z)$ will compensate $g_m(z)$, while $f_2(z)$ will compensate $g_{m-1}(z)$, and so on. Hence, the objective function (13) is modified to (16), while the total objective function (14) and the optimization problem (15) do not change.

$$h_j = \sum_{k=1}^{\frac{N}{2}} |(1 - f_j(z)g_{m+1-j}(z))q(z)|_{\omega=\omega_k}, \quad \forall\omega_k = \frac{2\pi k}{NT}. \qquad (16)$$

The proposed design can be applied for both minimum and non-minimum phase plant. However, it is still limited for a class of square MIMO model. This is due to the RGA analysis and MIMO stability assessment which require the calculation of the inverse of the matrix. For future research, the works can be extended to a non-square MIMO model.

4 Simulation results

Simulation is performed to validate the effectiveness of the proposed design. A 2×2 MIMO model of a 2 DOF robot is used. The MIMO model represents the plant used in the real-time experiments, and has the following transfer functions:

$$\boldsymbol{G}(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \qquad (17)$$

where

$$g_{11}(s) = \frac{1.021}{0.006s^2 + 0.119s + 1} \qquad (18)$$

$$g_{12}(s) = \frac{-0.014s + 0.397}{26.430s^2 + 7.202s + 1} \qquad (19)$$

$$g_{21}(s) = \frac{-0.003s}{0.007s^2 + 0.120s + 1} \qquad (20)$$

$$g_{22}(s) = \frac{1.003}{0.005s^2 + 0.115s + 1}. \qquad (21)$$

The RGA test (9) suggests that the dynamics of $g_{11}(s)$ and $g_{22}(s)$ give dominant interaction to output $y_1(t)$ and $y_2(t)$ respectively. Thus, the sub-compensator $f_{11}(z)$ and $f_{22}(z)$ are required to compensate $g_{11}(z)$ and $g_{22}(z)$, respectively.

Let the period of reference signal and the sampling period be 2 s and 0.025 s respectively. This gives the number of samples per reference period N as 80. The zero phase low pass filter $q(z)$, compensator order $(n_j)_{j=1,2}$, positive constants γ and ϵ, are chosen respectively as follows: $q(z) = 0.25z + 0.5 + 0.25z^{-1}$, $n_1 = 2$, $n_2 = 2$, $\gamma = 0.075$, and $\epsilon = 0.05$. Solving the optimization problem (15) by using optimization toolbox matlab results in the following compensator matrix:

$$\boldsymbol{F}(z) = \begin{bmatrix} f_{11}(z) & 0 \\ 0 & f_{22}(z) \end{bmatrix} \qquad (22)$$

where

$$f_{11}(z) = \frac{35.600z^2 - 56.910z + 24.340}{z^2 + 1.850z + 0.860} \tag{23}$$

$$f_{22}(z) = \frac{32.290z^2 - 50.310z + 21.110}{z^2 + 1.850z + 0.860}. \tag{24}$$

Equations (22)−(24) show that the obtained sub compensators have low order, stable and causal transfer functions. In the simulation, both channels are required to track triangular reference signals, where the reference signal of channel 2 is started 0.5 s after of channel 1 . The tracking outputs and errors are shown in Fig. 2, where the tracking errors vanish in about three repetitions. This shows the good tracking performance of the proposed compensator.

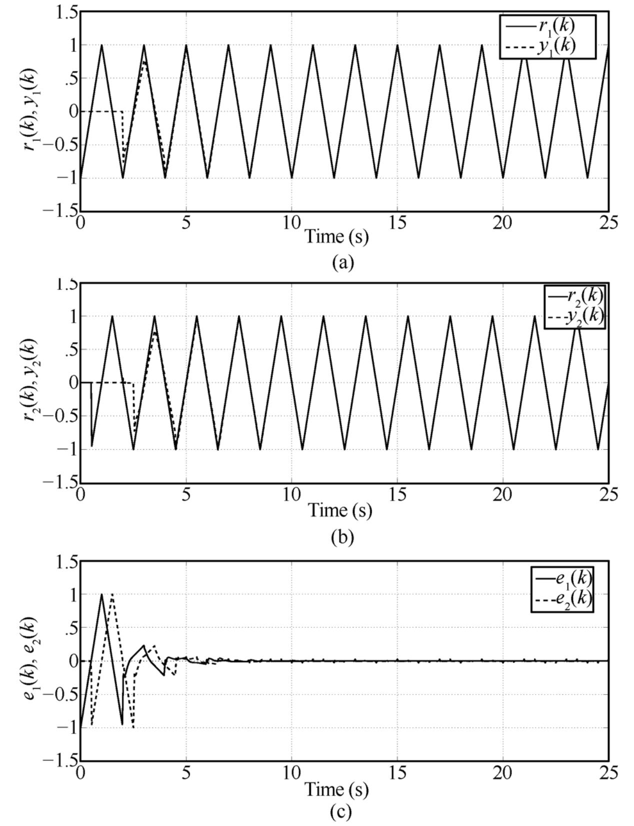

Fig. 2 Tracking results of the proposed compensator (a) tracking output $y_1(k)$, (b) tracking output $y_2(k)$, and (c) tracking errors $e_1(k), e_2(k)$

A comparison study is also given to show the significance of the proposed compensator. The comparison study is conducted with the PCI compensator proposed in [18]. The PCI compensator is based on the zero phase tracking error controller (ZPETC) of the SISO RC system proposed in [5], where the design aim is to perfectly cancel the phase response of the plant. The PCI is a matrix function such that[18]

$$\boldsymbol{F}_{pci}(z)\boldsymbol{G}(z) = \boldsymbol{I}_{m\times m}(z). \tag{25}$$

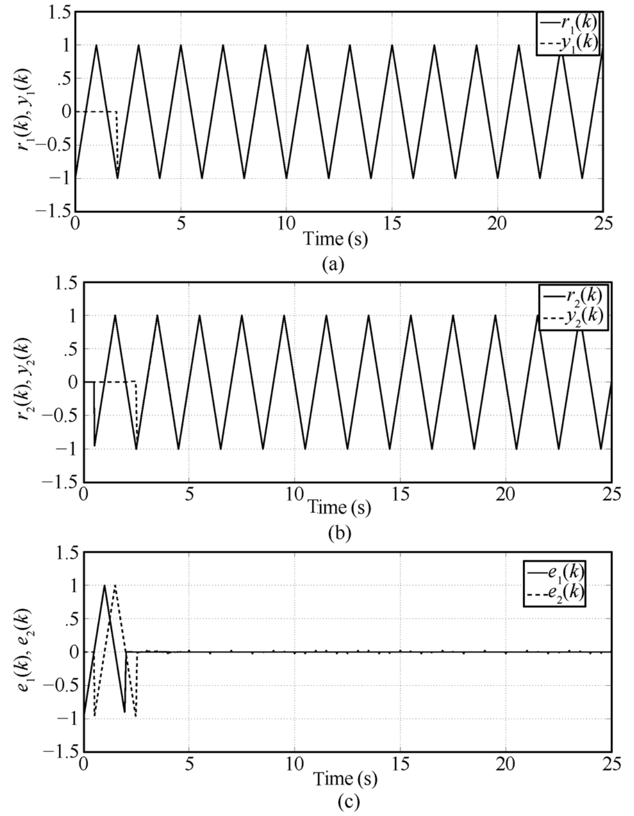

Fig. 3 Tracking results of the PCI compensator (a) tracking output $y_1(k)$, (b) tracking output $y_2(k)$, and (c) tracking errors $e_1(k), e_2(k)$

PCI compensators of stable minimum and non-minimum phase systems are formulated in (26) and (27), respectively

$$\boldsymbol{F}_{pci}(z) = Adj\,\boldsymbol{G}(z)\frac{1}{\det \boldsymbol{G}(z)} \tag{26}$$

$$\boldsymbol{F}_{pci}(z) = Adj\,\boldsymbol{G}(z)\frac{\beta^-(z^{-1})}{\beta^+(z)} \tag{27}$$

where $Adj\,\boldsymbol{G}(z)$ is the adjoint of $\boldsymbol{G}(z)$, $\beta^+(z^{-1})$ and $\beta^-(z)$ are stable and unstable part of $\det \boldsymbol{G}(z)$ respectively, $\beta^+(z^{-1})$ is $\beta^+(z)$ with operator z is replaced with the backward shift operator z^{-1}.

$$\boldsymbol{F}_{pci}(z) = \begin{bmatrix} f_{11}(z) & f_{12}(z) \\ f_{21}(z) & f_{22}(z) \end{bmatrix} \tag{28}$$

where

$$f_{11}(z) = \frac{21.940z^3 - 15.210z^2 - 14.390z + 10.970}{z^2 + 1.673z + 0.699} \tag{29}$$

$$f_{12}(z) = \frac{3.850z^5 - 14.320z^4 + 22.980^3 - 18.540z^2 + 7.520z - 1.230}{z^4 + 0.097z^3 - 1.290z^2 - 0.020z + 0.450} \tag{30}$$

$$f_{21}(z) = \frac{10^{-2}(0.380z^5 - 1.930z^4 + 3.660^3 - 3.360z^2 + 1.510z - 0.270)}{z^4 - 0.320z^3 - 1.640z^2 + 0.270z + 0.690} \quad (31)$$

$$f_{22}(z) = \frac{19.650z^3 - 12.390z^2 - 13.330z + 9.440}{z^2 + 1.673z + 0.699}. \quad (32)$$

To design a PCI compensator for the plant model in (17), we need to examine the zeros and poles of det $\boldsymbol{G}(z)$. The det $\boldsymbol{G}(z)$ has 6 zeros and 8 poles, where all zeros and poles are inside the unit circle. Thus, the discrete plant $\boldsymbol{G}(z)$ is stable minimum phase system. The obtained PCI compensator matrix is shown in (28)−(32).

We can see that PCI based RC has 4 sub compensators, and two of them have the order as high as 5, while the proposed compensators have only 2 sub compensators and both have the order of 2. Moreover, all PCI based RCs are non-causal, which cannot be stand-alone, and have to be combined with the internal model as illustrated in the equation below:

$$\boldsymbol{G}_{rc}^{pci}(z) = \begin{bmatrix} \frac{q(z)}{z^N - q(z)} f_{11}(z) & \frac{q(z)}{z^N - q(z)} f_{12}(z) \\ \frac{q(z)}{z^N - q(z)} f_{21}(z) & \frac{q(z)}{z^N - q(z)} f_{22}(z) \end{bmatrix}. \quad (33)$$

Fig. 3 shows that the tracking outputs of PCI follow the trajectories after one repetition, while the tracking outputs of the proposed compensator shown in Fig. 2 take about three repetitions. This shows the superiority of the PCI in terms of the convergence rate. This is due to the unity gain of $(\boldsymbol{F}_{pci}(z)\boldsymbol{G}(z))$. The unity gain results in uniform convergence, where the tracking error vanishes within one cycle[18]. The proposed compensator only approximates the inverse of strong dynamic elements in the plant model. This makes the unity gain of $(\boldsymbol{F}_{pci}(z)\boldsymbol{G}(z))$ hard to achieve. In terms of complexity, the proposed compensator is simpler than the PCI as it only has m sub compensators. Moreover, all sub compensators are in low order, stable, and causal form. The complexity of the PCI design will arise when the plant model is of the high order and the choice of sampling period is very small.

5 Experimental results

This section presents the experimental results of the proposed compensator. The real-time experiments are conducted on the 2 DOF Quanser robot plant pictured in Fig. 4. Fig. 5 shows a set of Quanser hardware used in the experiments. Two servo motors mounted at a fixed distance control two arms coupled via two non-powered two-link arms. The system has 2 actuated and 3 unactuated revolute joints. The 4-bar linkage system gives coupling effect to the actuated joints. The 2 DOF Quanser robot is a 2×2 MIMO system, and its transfer functions are experimentally modeled using system identification toolbox of matlab, and given in (17)−(21).

The experiments aim to control the end effector (joint E) to have diamond shape movement. This can be done by giving a triangular reference signal at each channel. The reference signals are in $X-Y$ Cartesian coordinates (E_{xd}, E_{yd}), where E_{xd} and E_{yd} values are fed to Channels 1 and 2, respectively. The period of both E_{xd} and E_{yd} are 2 s, and E_{yd} is started 0.5 s after E_{xd}. The proposed compensator is used to control the position of the end effector. The tracking outputs and errors of the system are shown in Fig. 6, while the end-effector $X-Y$ position response is shown in Fig. 7.

Fig. 6 shows that the tracking errors in both channels converge to zero after 3 repetitions. Figs. 2 (c) and 6 (c) indicate that the tracking errors between simulation and experimental results show similar transient behavior, where they converge after 3 repetitions. However, the tracking errors at steady state are different as shown in Fig. 8. Fig. 8 shows that the tracking error from the experiment has larger amplitude and more noisy compared to the tracking error from the simulation.

Fig. 7 shows the trace of end-effector E in inches after reaching a steady state. It can be seen that the trace (red line) forms a diamond shape, and it accurately follows the set point (blue line). This tracking performance verifies the effectiveness of the proposed design, and also validates the simulation results presented in the previous section.

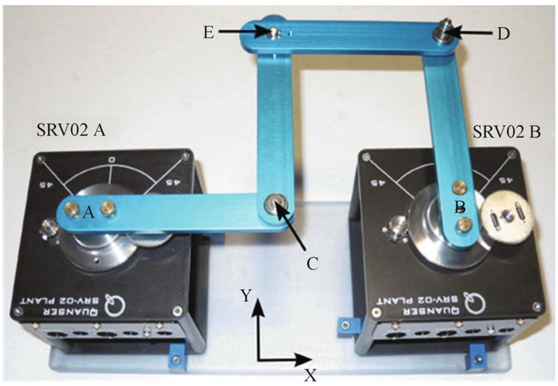

Fig. 4 2 DOF Quanser robot plant[32]

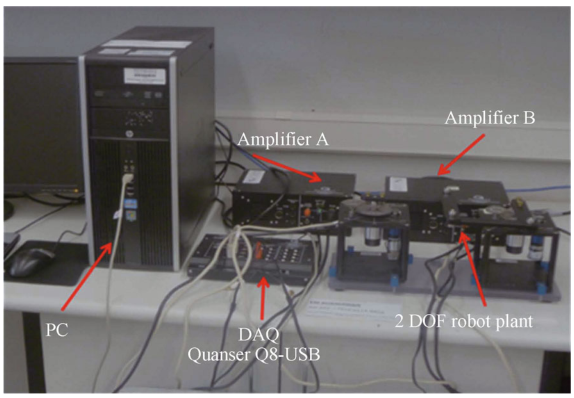

Fig. 5 Experimental system hardware setup

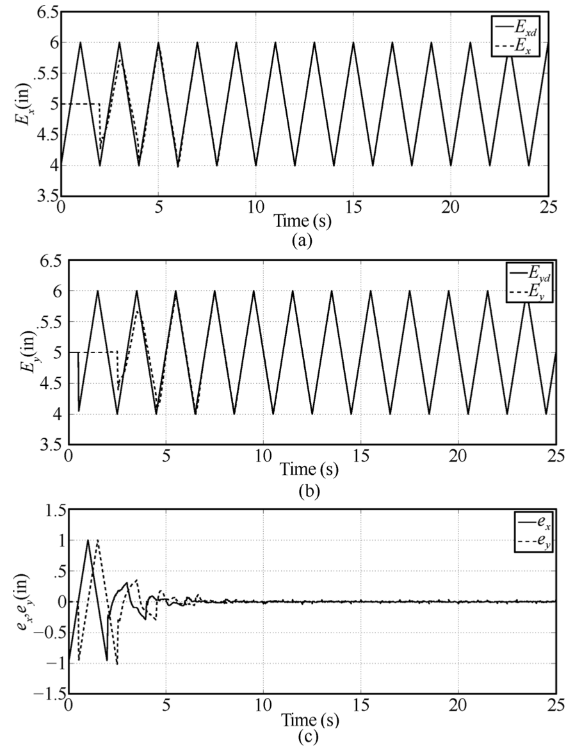

Fig. 6 Tracking output: (a) $E_x(k)$; (b) $E_y(k)$; (c) $e_x(k)$ and $e_y(k)$.

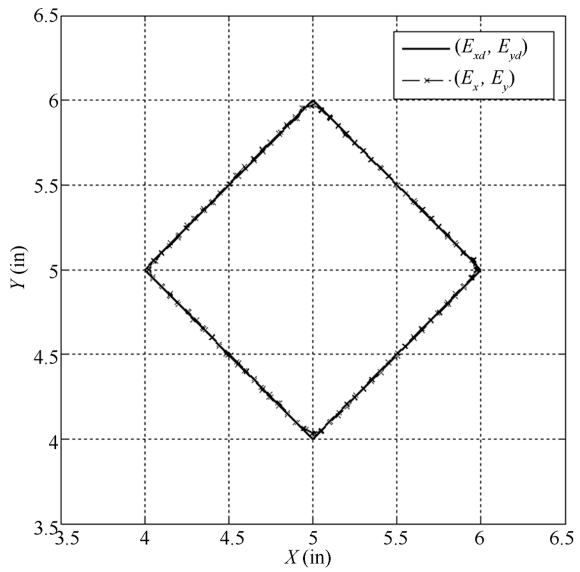

Fig. 7 End effector $X - Y$ position response of the proposed compensator

6 Conclusions

This paper presents compensator design for MIMO RC system based on decentralized control. Relative gain array (RGA) analysis was used initially to obtain the best pairing of inputs and outputs. Then, the compensator matrix that guarantees the stability of MIMO RC system, was designed using optimization to obtain low order, stable and causal sub compensators. The proposed compensator has been verified by simulation and real-time experiments.

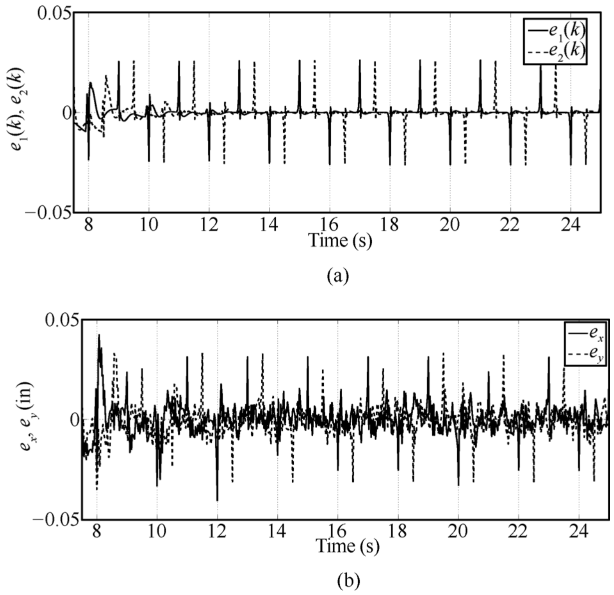

Fig. 8 Steady state tracking error from (a) the simulation and (b) the experiment.

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