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New Time-varying Fuzzy Sets Based on a PSO Midpoint of the Universe of Discourse

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Abstract: The paper presents a robust parallel distributed compensation (PDC) fuzzy controller for a nonlinear and certain system in continuous time described by the Takagi-Sugeno (T-S) fuzzy model. This controller is based on a new type of time-varying fuzzy sets (TVFS). These fuzzy sets are characterized by displacement of the kernels to the right or left of the universe of discourse, and they are directed by a well-defined criterion. In this work, we only focused on the movement of midpoint of the universe. The movements of this midpoint are optimized by particle swarm optimization (PSO) approach.

Keywords: Fuzzy sets, fuzzy system, membership function, parallel distributed compensation (PDC) fuzzy controller, particle swarm optimization (PSO) approach, linear matrix inequality (LMI).

1 Introduction

The interest in takagi-sugeno (T-S) model[1, 2] is due to the fact that the stability and performance characteristics of the system can be analyzed using a Lyapunov function approach. It achieved great success in its application to many systems in real world[3−5]. The so-called parallel distributed compensation (PDC)[2, 6, 7] utilizes a nonlinear state feedback controller which mirrors the structure of associated T-S model. The stability analysis is based on the quadratic Lyapunov function[7,8], which is easy to implement and can be expressed as a convex optimization problem in linear matrix inequality (LMI) formalism[7−11].

Fuzzy controller′s design depends mainly on the rules based of fuzzy sets and membership functions, which contain the linguistic elements who characterize the functioning of the industrial process. In reality, we cannot exactly evaluate the length of an element of fuzzy sets. For example, temperature′s linguistic variables are "Low", "Medium" and "High". These linguistics values of fuzzy sets do not have a well-defined numeric range at all the time and they also depend on the process. In general, we approximate the linguistics values of fuzzy sets by a proper numeric range, where the membership functions are fixed during the computation time called fixed fuzzy sets (FFS). Conventionally, this type of fuzzy sets is known as type-1 fuzzy sets. The type-2 fuzzy set is a set where we also have uncertainty on the membership function[12, 13].

In context of the self organizing fuzzy control (SOFC)[14−25], this work is an extension of [22, 23] which were interested in membership functions by proposing that the ranges of linguistics values of the fuzzy sets vary during the computation time, called the time-varying fuzzy sets (TVFS). In general, movements of all support points are performed in both directions, i.e., left or right and with variable distances. Our approach is to only adjust the midpoint of the universe of FFS by a function depending on the error. The displacement of the midpoint is defined using a temporal function and is directed to both right and left sides of the universe by the position of the premise variable. All other support points of the fuzzy sets are shifted by the same function and direction except that the two extreme support points are fixed for all time.

Salim et al.[22, 23] deal with identifying the parameters of TVFS method. Rererence [22] discusses an offline identification based on recursive least squares (RLS), where the parameters are fixed during the execution time, reference [23] is an online identification based on an adaptive approach, where the parameters vary in real-time. In this paper, we propose a new online identification based on optimization using particle swarm optimization (PSO) approach.

By applying this TVFS for designing a PDC fuzzy controller for nonlinear system, we use a decay rate controller and relaxed stability conditions[26−28]. A sample system (inverted pendulum) is given to show the robustness of the PDC fuzzy controller based on this approach.

The paper is organized as follow. The T-S fuzzy model and stability using Lyapunov approach and PDC fuzzy controller are recalled in Section 2. Section 3 discusses the time-varying fuzzy sets. Section 4 presents the design of the algorithm. A simulation example is provided to show the effectiveness of this approach in Section 5. Finally, conclusions are given in Section 6.

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2 Fundamentals

2.1 T-S fuzzy model

Consider a nonlinear system described by the T-S fuzzy model[1,2]:

Plant rule i : IF $z_1(t)$ is M_{i1}, $\cdots$, and $z_p(t)$ is M_{ip}, THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t),\ i = 1, \cdots, r \end{cases} \tag{1}$$

where M_{ij} is the fuzzy set, and r is the number of IF-THEN rules, $z(t) = [z_1(t), z_2(t), \cdots, z_p(t)]$ are the premise variables, $A_i \in \mathbf{R}^{n\times n}$, $B_i \in \mathbf{R}^{n\times m}$ and $C_i \in \mathbf{R}^{n\times 1}$ are system matrices where $m \le n, x \in \mathbf{R}^n$ is the state, $u \in \mathbf{R}^m$ is the control constrained as

$$\| u(t) \|_2 < \varphi. \tag{2}$$

The output $y(t)$ is constrained as

$$\| y(t) \|_2 < \rho. \tag{3}$$

The considered fuzzy model can be written as

$$x(t) = \frac{\sum\limits_{i=1}^{n} w_i(A_i x(t) + B_i u(t))}{\sum\limits_{i=1}^{n} w_i(t)} \tag{4}$$

where w_i is defined as

$$w_i(z(t)) = \prod_{j=1}^{r} M_{ij}(z_j(t)) \tag{5}$$

M_{ij} is membership function of the j-th fuzzy set in the i-th rule. Let us define

$$h_i(z(t)) = \frac{w_i(z(t))}{\sum\limits_{i=1}^{r} w_i(z(t))} \tag{6}$$

$$\begin{cases} \sum\limits_{i=1}^{r} (z(t)) = 1 \\ h_i(z(t)) \ge 0,\ i = 1, \cdots, r \end{cases} \tag{7}$$

for every input $x(t)$ and $u(t)$, the global output is obtained by

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{r} h_i(z(t))\{A_i x(t) + B_i u(t)\} \\ y(t) = \sum\limits_{i=1}^{r} h_i(z(t))\{C_i x(t)\},\ i = 1, \cdots, r \end{cases} \tag{8}$$

where matrices A_i and B_i are constants of appropriate size and satisfy the assumption that: each pair (A_i, B_i) is stabilizable.

2.2 Parallel distributed compensation controller

To stabilize the system represented by (6), we use a PDC controller defined by[2, 6, 7]. Control rule i: IF $z_1(t)$ is M_{i1} and , $\cdots$, and $z_p(t)$ is M_{ip} THEN

$$u(t) = -K_i x(t)\ \ i = 1, \cdots, r \tag{9}$$

where K_i is the controller stabilizing the i-th subsystem. The global control will be given by

$$u(t) = -\frac{\sum\limits_{i=1}^{r} w_i(z(t))K_i.x(t)}{\sum\limits_{i=1}^{r} w_i(z(t))} = -\sum_{i=1}^{r} h_i(z(t))K_i x(t),\ i = 1, \cdots, r. \tag{10}$$

2.3 Quadratic stability via Lyapunov approach

For guaranteeing the synthesizable fuzzy controller stability, we use the theorems giving the sufficient conditions of Lyapunov quadratic stability which exploits LMI formalism[29−32].

3 Time-varying fuzzy sets

3.1 Definition and presentation of the TVFS

On the fuzzy sets form, we have no confusion to give the two extreme numeric ranges of their corresponding linguistics values. Around the midpoint of the universe, there is always a wide margin for intersection of linguistics values where we cannot determine their exact numeric ranges fixed throughout the computation time. We propose that the ranges of the fuzzy sets vary with time on the universe $[\underline{Z}, \overline{Z}\ ]$, called time-varying fuzzy sets.

These linguistics ranges are inversely proportional to membership grades (Figs. 1−3) which are directly proportional to the control law, defined by the PDC fuzzy controller. Let us define the error $e(t)$ as

$$e(t) = x(t) - x_d(t) \tag{11}$$

where $x(t)$ is the current system state and $x_d(t)$ is the desired system state.

If $e(t)$ is large, then one needs more control effort to decrease when approaching the desired state by an adequate acceleration. For example, consider the fuzzy set Low or High, if the range of one decreases by one step, the other range increases by the same step.

To carry out this objective, we propose to adjust the midpoint of the universe defined by α in Fig. 1. The displacements of this midpoint are characterized by a continuous function depending on the error[23], where

$$\alpha(t) = \frac{f(e(t))}{\alpha(t)} \in [\underline{\alpha}, \overline{\alpha}] \subset [\underline{Z}, \overline{Z}]. \tag{12}$$

Fig. 3 presents a case where $\alpha(t)$ is displaced to the left side, then the membership grade of the $z_2(t)$ premise variable is considerably increased, while those of the $z_1(t)$ premise variable is slightly decreased.

Through Fig. 1, if the midpoint is shifted towards the left $\{\alpha(t_0), (\alpha(t_1), \cdots, (\alpha(t_i)\}$, the decrease of the left range, it causes a higher membership grades on the left side

$\{\mu_{z1}(t_0) < \mu_{z1}(t_1) < \cdots < \mu_{z1}(t_i)\}$ and the increase of these membership grades generates an increase of the control effort, see (10).

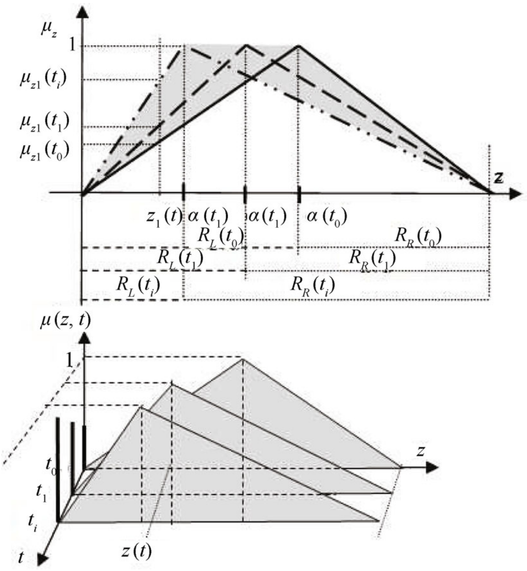

Fig. 1 Membership functions with the time-varying midpoint (2D and 3D)(μ: membership grade, Z: premise variable, $R_R(t)$: right range, $R_L(t)$: left range, $\alpha(t)$: midpoint)

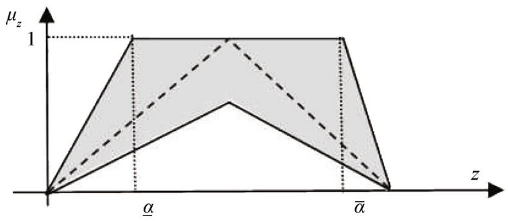

Fig. 2 Area of membership functions covered by the time-varying midpoint: called Footprint of Shifting (FOS)

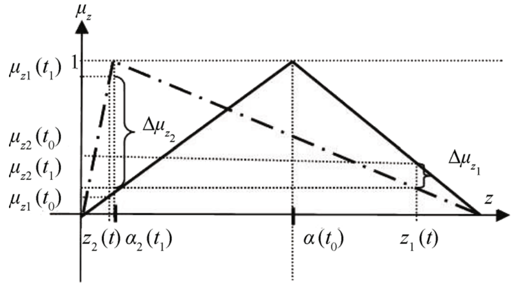

Fig. 3 Variation of the membership grades generated by the time-varying midpoint

3.2 Proof relation between the control and the grades membership function

From the (5) that

$$h_i = \frac{w_i}{\sum\limits_{i=1}^{r} wi} = \frac{wi}{w_1 + w_2 + \cdots + w_i + \cdots + w_r} \geq 0 \quad (13)$$

$$W = w_1 + w_2 + \cdots + w_{i-1} + w_{i+1} \cdots + w_r > 0. \quad (14)$$

For $w_i{}'$, we have

$$h_i{}' = \frac{w_i{}'}{W + w_i{}'}. \quad (15)$$

Then,

$$h_i{}' - h_i = \frac{w_i{}'}{W + w_i{}'} - \frac{w_i}{W + w_i} = \frac{W}{(W + w_i) \times (W + w_i{}')} \times (w_i{}' - w_i). \quad (16)$$

So,

$$\text{if } w_i{}' - w_i > 0, \text{ then} \quad h_i{}' - h_i > 0. \quad (17)$$

So increase of w_i generates an increase in h_i, and from (10), the increase of h_i generates an increase in the control effort $u(t)$.

4 Algorithm design for TVFS

We propose five steps

1) The design (inclusion) of $\alpha(t)$ midpoint into the algorithm of fuzzy system

2) The functions defining the $\alpha(t)$ displacements

3) The direction criterion of the displacement of the $\alpha(t)$ midpoint

4) The effect of the $\alpha(t)$ function on the stability

5) Identification of the parameters of $\alpha(t)$.

We explain this algorithm using triangular membership functions and PDC fuzzy controller.

4.1 Design of fuzzy system by including $\alpha(t)$ midpoint into the algorithm

Let us consider triangular membership functions in Fig. 4. The premise variable $z_i(t)$ is given by

$$z_i(t) = M_{1i}(z_i(t)) \times \overline{z}_i + M_{2i}(z_i(t)) \times \underline{z}_i \quad (18)$$

$[\overline{z}_i, \underline{z}_i]$ represent minimum and maximum of z_i for $x(t) \in [\overline{x}, \underline{x}]$.

M_{1i} and M_{2i} are the membership grades of $z_i(t)$ with α fixed, as shown in Fig. 4.

$$M_{1i}(z_i(t)) = \frac{z_i(t) - \underline{z}_i}{\overline{z}_i - \underline{z}_i}$$

$$M_{2i}(z_i(t)) = \frac{\overline{z}_i - z_i(t)}{\overline{z}_i - \underline{z}_i}. \quad (19)$$

The value of α is fixed as the midpoint of the universe.

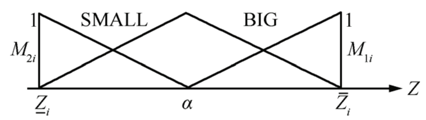

Fig. 4 Membership functions with α fixed

And we propose to calculate the premise variable z_i(t) by

$$z_i(t) = M_{1i}(z_i(t))\alpha(t) + M_{2i}(z_i(t))\underline{z}_i + M_{3i}(z_i(t))\overline{z}_i + M_{4i}(z_i(t))\alpha(t) \quad (20)$$

where

$$\alpha(t) \in [\underline{\alpha}\,, \overline{\alpha}] \subset [\underline{Z}\,, \overline{Z}] \tag{21}$$

$$\begin{aligned} M_{1i}(z_i(t)) &= \frac{1}{2} \times \frac{z_i(t) - \underline{z}_i}{\alpha(t) - \underline{z}_i} \\ M_{2i}(z_i(t)) &= \frac{1}{2} \times \frac{\alpha(t) - z_i(t)}{\alpha(t) - \underline{z}_i} \\ M_{3i}(z_i(t)) &= \frac{1}{2} \times \frac{z_i(t) - \alpha(t)}{\overline{z}_i \alpha(t)} \\ M_{4i}(z_i(t)) &= \frac{1}{2} \times \frac{\overline{z}_i - z_i(t)}{\overline{z}_i \alpha(t)} \end{aligned} \tag{22}$$

M_{1i}, M_{2i}, M_{3i} and M_{4i} are the membership grades of $z_i(t)$ with α time-varying.

4.2 Functions defining the $\alpha(t)$ displacements

Let $\alpha_R(t)$ be the right displacement of the midpoint on the universe, ensured by a function depending on the error, as shown in Fig. 5.

$$\alpha_R(t) = f_R(e(t)) = \theta_{1R} \times \left(1 - \mathrm{e}^{-\theta_{2R} \times |e(t)|}\right) \tag{23}$$

where θ_{1R} and θ_{2R} are the maximum and the growth rate of $\alpha_R(t)$, respectively.

$\alpha_L(t)$ is the left displacement of the midpoint on the universe, ensured by

$$\alpha_L(t) = f_L(e(t)) = -\theta_{1L} \times \left(1 - \mathrm{e}^{-\theta_{2L} \times |e(t)|}\right) \tag{24}$$

where $-\theta_{1L}$ and θ_{2L} are the minimum and the decay rate of the $\alpha_L(t)$, respectively.

4.3 Direction criterion displacement of the $\alpha(t)$ midpoint

The direction criterion depends on the relation between the membership grades and the control law based on the error distance $e(t)$, see Section 3.1. In this note, we use a PDC controller and we based it on the relationship (10). The displacement of the $\alpha(t)$ midpoint to both left or right is directed by the position of the premise variable $Z(t)$, i.e., if the premise variable is set to the left of the midpoint, then $\alpha(t)$ must approach to the minimum $\underline{\alpha}$ by the function defined in relationship (24). If the premise variable is set to the right of the midpoint, then $\alpha(t)$ must approach to the maximum $\overline{\alpha}$ by the function defined in relationship (24). The relationship (21) is always checked.

So, we propose that the displacements $\alpha(t)$ will follow the $Z(t)$ premise variable by an acceleration determined by the output reference of the Mamdani fuzzy model, shown in Figs. 6 and 7. The switch function between the right and the left displacements is illustrated in Fig. 5, and is given by this sub-program:

$$\begin{aligned} &\alpha(t) < z(t+1) \text{ then } \alpha(t+1) = \alpha_R(t+1) \\ &\qquad \text{else} \quad \alpha(t+1) = \alpha_L(t+1). \end{aligned} \tag{25}$$

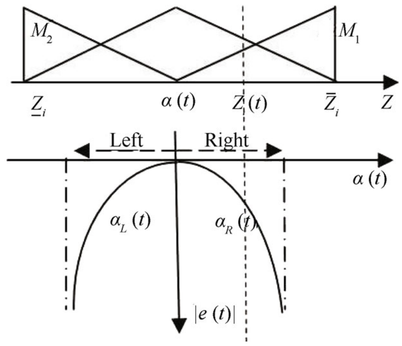

Fig. 5 Direction of the displacement of the time-varying midpoint

4.4 Effect of $\alpha(\mathbf{t})$ functions on the stability

It is simple to demonstrate that (18) and (20) are exactly equal for any value of $\alpha(t)$, by substituting respectively equation (19) in (18) and (22) in (20). This implies that the subsystems $[A_i\ B_i;\ C_i\ 0]$ of T-S[2, 7] do not change as the value of $\alpha(t)$, and also the criteria of the stability (Stability theorems[29−32]) do not change.

4.5 Identification of $\alpha(\mathbf{t})$ parameters

In this part, we use an online identification of $\alpha(t)$ function shown by Fig. 6 based on the traditional model reference adaptive control (MRAC).

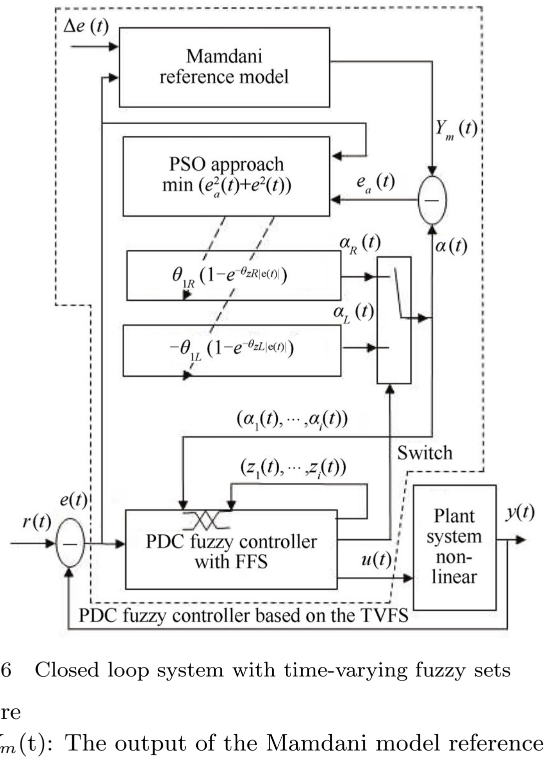

Fig. 6 Closed loop system with time-varying fuzzy sets

where

$Y_m(t)$: The output of the Mamdani model reference

$e(t)$: Error

$\triangle e(t)$: Change in error

$e_a(t)$: Adaptive error

$r(t)$: Reference

$u(t)$: Control law

$\alpha_L(t)$: The left displacement function of the midpoint

$\alpha_R(t)$: The right displacement function of the midpoint
$\alpha(t)$: The general displacement function of the midpoint
$[z_1(t), \cdots, z_n(t)]$: Premise variable vector
$[\alpha_1(t), \cdots, \alpha_i(t)]$: The vector of the midpoint function
$y(t)$: Output of the system.

4.5.1 Reference model

The reference model is defined by the output $Y_m(t)$ of Mamdani fuzzy system, which represents the acceleration of the midpoint displacements presented in Fig. 7.

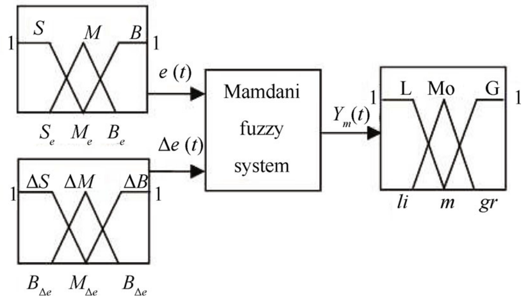

Fig. 7 Representation of the reference model

4.5.2 Rule-base of $\alpha(t)$ displacements

As an example we give in Table 1 the rule-base of the system.

Table 1 Rule base

$e(t)/\triangle e(t)$	Small	Medium	Big
$\triangle S$	Little	Moderate	Great
$\triangle M$	Little	Moderate	Great
$\triangle B$	Moderate	Great	Great

4.5.3 Approach of the particle swarm optimization

The PSO method is a member of wide category of swarm intelligence methods for solving the optimization problems, which is a population based search algorithm where each individual is referred to as particle and represents a candidate solution. Each particle in PSO flies through the search space with an adaptable velocity that is dynamically modified according to its own flying experience and also to the flying experience of the other particles. In PSO, all particles strive to improve themselves by imitating traits of their successful peers. Further, each particle has a memory and hence it is capable of remembering the best position in the search space ever visited by it. The position corresponding to the best fitness of one particle is known as *pbest* and the overall best out of all the particles in the population is called *gbest*[33, 34].

The velocity and the position of each particle can be calculated using the current velocity and the distances from the $pbest_{j,g}$ to $gbest_g$ as shown in the following formulas:

$$v_{j,g}^{(ite+1)} = I_w \times v_{j,g}^{(ite)} + c_1 \times r_1 \times (pbest_{j,g} - x_{j,g}^{(ite)}) + c_2 \times r_2 \times (gbest_g - x_{j,g}^{(ite)}) \quad (26)$$

$$x_{j,g}^{(ite+1)} = x_{j,g}^{(ite)} + v_{j,g}^{(ite+1)} \quad (27)$$

where $j = 1, 2, \cdots, n$, $g = 1, 2, \cdots, m$, n is the number of particles in the swarm, m is the number of components for the vector v_j and x_j, *ite* is the number of iterations (generations), $v_{j,g}^{(ite)}$ is the g-th component of the velocity of the particle j at iteration t, Iw is the inertia weight factor, c_1 and c_2 are the cognitive and social acceleration factors, r_1 and r_2 are the random numbers uniformly distributed in the range [0, 1], $x_{j,g}^{(ite)}$ is the g-th component of the position of particle at iteration t, $pbest_j$ is the *pbest* of particle j, $gbest_g$ is the *gbest* of group.

The j-th particle in the swarm is represented by a d-dimensional vector $x_j=(x_{j,1}, x_{j,2}, \cdots, x_{j,d})$ and its rate of position change (velocity) is denoted by another d-dimensional vector $v_j=(v_{j,1}, v_{j,2}, \cdots, v_{j,d})$. The best previous position of the j-th particle is represented as $pbest_j=(pbest_{j,1}, pbest_{j,2}, \cdots, pbest_{j,d})$. The index of best particle among all of the particles in the swarm is represented by the $gbest_g$. In PSO, each particle moves in the search space with a velocity according to its own previous best solution and its group′s previous best solution. The velocity update in a PSO consists of three parts, namely momentum, cognitive and social parts (27). The balance among these parts determines the performance of a PSO algorithm. The parameters c_1 and c_2 determine the relative pull of *pbest* and *gbest* and the parameters r_1 and r_2 help in stochastically varying these pulls. In the above equations, superscripts denote the iteration number. Fig. 8 shows the velocity and the position in two-dimensional parameter space.

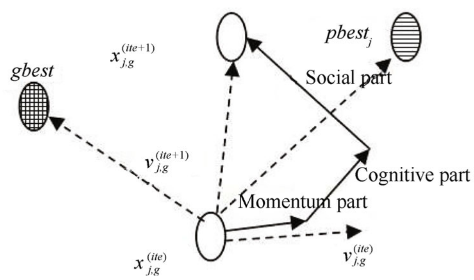

Fig. 8 Description of velocity and position updates in PSO for 2-dimensional parameter space

Based on the closed loop system with TVFS illustrated in Fig. 6, we have

$$e_a(t) = \alpha(t) - Y_m(t). \quad (28)$$

The general form of $\alpha(t)$ midpoint function is given by

$$\alpha(t) = \theta_1(1 - \mathrm{e}^{-\theta_2 \cdot e(t)}). \quad (29)$$

So, the objective of using the PSO approach is to find the parameters $\theta_1(t)$ and $\theta_2(t)$ such that the objective function J is optimized which is defined by

$$J = \min\left(e_a^2(t) + e^2(t)\right) = \min\left[(\alpha(t) - Y_m(t))^2 + e^2(t)\right] = \min\left[\left(\theta_1(1 - \mathrm{e}^{-\theta_2 e(t)}) - Y_m(t)\right)^2 + (x(t) - x_d(t))^2\right]. \quad (30)$$

5 Simulation example

To illustrate the idea of this note, we consider the problem of an inverted pendulum on a cart[2]:

$$\begin{cases} x_1(t) = x_2(t) \\ x_2(t) = \dfrac{g\sin(x_1(t)) - a \times m \times x_2^2(t) \times \frac{\sin(2x_1(t))}{2} - h(t)}{\frac{4l}{3} - a \times m \times l\cos^2(x_1(t))} \\ h(t) = a \times \cos(x_1(t)) \times u(t). \end{cases} \tag{31}$$

We approximate the system by the following two-rule fuzzy model:

$$A_1 = \begin{bmatrix} 0 & 1 \\ \dfrac{g}{4lm - aml} & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ \dfrac{a}{\frac{4lm}{3} - a \times m \times l} \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ \dfrac{2 \times g}{\pi(4 \times \frac{l}{3} - a \times m \times l\gamma^2)} & 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 \\ \dfrac{-a\gamma}{4 \times \frac{l}{3} - a \times m \times l \times \gamma^2} \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$x(0) = \begin{bmatrix} \dfrac{\pi}{4} \\ 0 \end{bmatrix}.$$

The membership functions are shown in Fig. 4, where $\frac{\pi}{2} \leq x_1(t) \leq \frac{\pi}{2}$.

The rule-base of the reference Mamdani model is in Table 1.

We can define the parameters of the membership function of the error and the change in error as

$$S_e = 1\% \times \mid x_1(0) \mid = 0.5^\circ$$
$$M_e = 3\% \times \mid x_1(0) \mid = 1.5^\circ$$
$$B_e = 6\% \times \mid x_1(0) \mid = 3.0^\circ$$
$$S_{\triangle e} = 1\% \times S_e$$
$$M_{\triangle e} = 3\% \times M_e$$
$$B_{\triangle e} = 6\% \times B_e.$$

Acceleration of the displacement is dependedent on the universe, we can take the maximum of the range: $li = 0$, $mo = 0.785$, $gr = 1.57$.

For PSO simulation parameters, we can use n=49 as the number of particles in swarm, ite=100 as the number of iterations, $c_1 = c_2 = 2$, $r_1 = r_2 = random[0, 1]$, $I_w = w_{\max} \times \frac{(w_{\max} - w_{\min})}{iter_{\max}}$, and iter is the inertia weight factor[33], $\theta_1 = 1, \theta_2 = 0, v_1 = 1, v_2 = 0$, initial parameters. The responses obtained are as

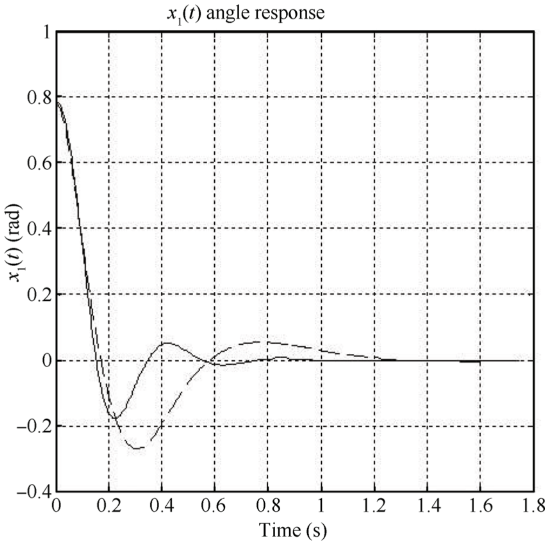

Fig. 9 x_1(t) response (—TVFS, −− FFS)

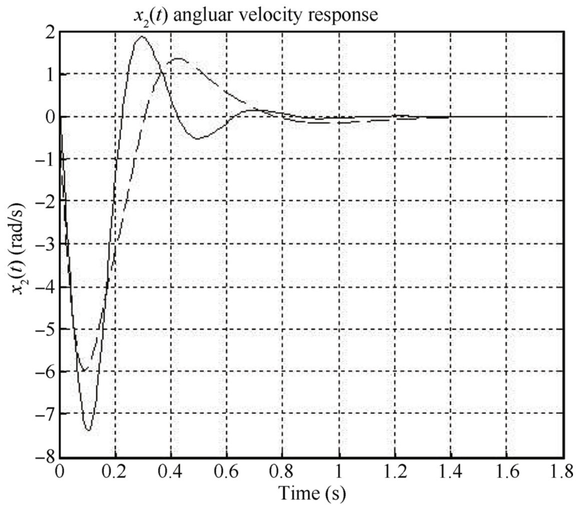

Fig. 10 x_2 (t) response (—TVFS, −− FFS)

5.1 Comments and comparison

The fuzzy system with the VFS approach gives a good stability and the dynamic specifications are better than the system with the FFS. Our results are improved compared to the results found in [2, 20], particularly the settling time in [14, 20]. We can see also that TVFS completely reduces oscillations but the control effort is relatively high compared to the FFS. We can observe that the effect of $\alpha(t)$ function is in the transient but later is got stabilized towards the final stability midpoint, which depends on the reference system, as shown in Fig. 7.

In the end, we define the footprint of shifting (FOS), which represents the area covered by the variation of fuzzy sets, which itself is an interval-valued fuzzy set (Fig. 13). The initial $\alpha(0)$ is set to the midpoint of the universe, through the response of the Fig. 12, we can compute the universe of $\alpha(t)$ as

$$FOS = \overline{\alpha} + \mid \underline{\alpha} \mid = 1.32 + 1.17 = 2.49 \in [-\frac{\pi}{2}, \frac{\pi}{2}]. \tag{32}$$

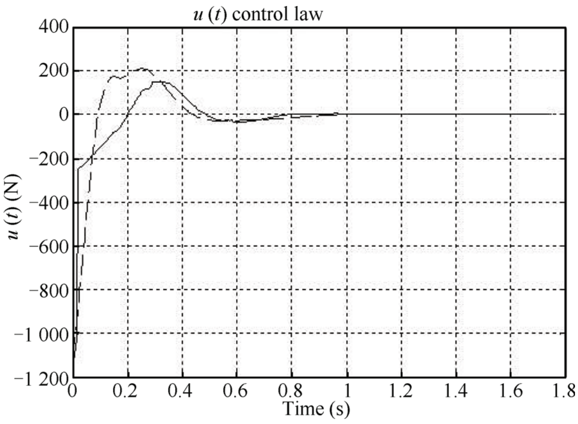

Fig. 11 Control effort response (—TVFS,−− FFS)

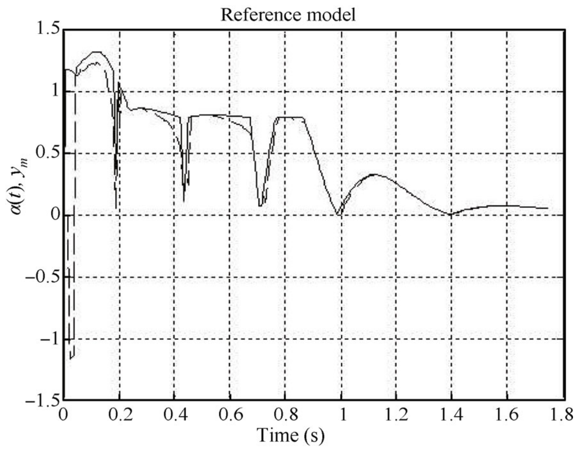

Fig. 12 $\alpha(t)$ response (−−) and Y_m reference (—)

By the PSO approach, we get a large FOS compared with the adaptive approach[23], provided that control effort and time of application is enough. This is explained by the good results, especially the settling time.

The FOS of the PSO approach is illustrated by Fig. 13 which is generated by the program.

6 Conclusions

The presented TVFS approach is based only on the distance of the error. The role of the $\alpha(t)$ midpoint function is to ensure an adequate effort and time required to apply this effort. Generally, it requires a high effort which accurately reflects that the time-varying fuzzy sets have given the necessary power and enough time to the controller for stabilizing the system.

In addition, the TVFS approach is built on the rule-base that defines the displacement functions of the midpoint, where it is very important to give a high or low control amplitude, and on the criterion that defines the direction of shifting, that is to increase (decrease) the membership grades that generate the increasing (decreasing respectively) the control effort. The displacement functions of the midpoint can cause oscillations of the control (cattering). For reducing these oscillations, we focus on a well defined rule-base (that varies quickly or slowly) and on the choice of the direction criterion. Through the TVFS approach, the ranges of the linguistic values of the fuzzy sets change with time according to the variation of the linguistic values of the error.

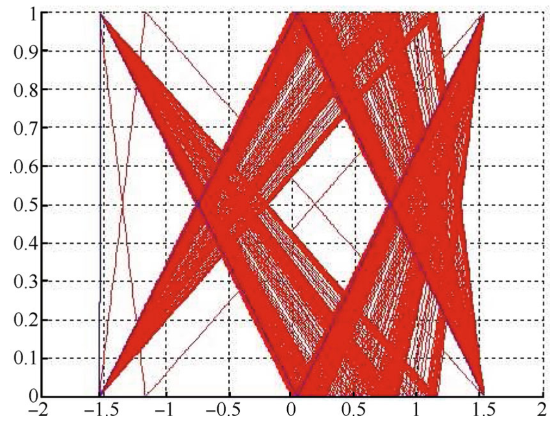

Fig. 13 FOS of VFS membership function

Finally, we can summarize this conclusion as Table 2.

Table 2 Comparison between three fuzzy sets

Type of fuzzy sets	Vertical interval *FOU*	Horizontal interval *FOS*
Type-1 fuzzy set	$FOU = 0, \forall t$	$FOS = 0, \forall t$
Type-2 fuzzy set	$FOU \neq 0, \forall t$	$FOS = 0, \forall t$
VFS	$FOU \neq 0, \forall t$	$FOS \neq 0, \forall t$

Appendix

Theorem 1.

Decay rate controller design using relaxed stability conditions: The condition that $\dot{V}(x) + 2\beta V(x(t)) \leq 0$ for all trajectories is equivalent to[28−31]

$$\exists P > 0, \exists Q > 0 \ \ G_{ii}^{\mathrm{T}} P + P G_{ii} + (s-1)Q + 2\beta P < 0, \beta > 0 \tag{A1}$$

$$\left(\frac{G_{ij}+G_{ji}}{2}\right)^{\mathrm{T}} P + P\frac{G_{ij}+G_{ji}}{2} - Q + 2\beta P \leq 0 \tag{A2}$$

$i < j$, s.t. $h_i \cap h_j \neq \phi$, where $1 < s < r$.

We can find the K_i controller by this optimization problem:

$$\max_{(X,Z,Y_1,\cdots,Y_r)} \beta$$
$$\begin{cases} -XA_i^{\mathrm{T}} - A_iX + B_iY_i + Y_i^{\mathrm{T}}B_i^{\mathrm{T}} - (s-1)Z- \\ \quad 2\beta X > 0, \quad i = 1, 2, \cdots, r \\ -XA_i^{\mathrm{T}} - A_iX - XA_j^{\mathrm{T}} - A_jX + B_iY_j + \\ \quad Y_j^{\mathrm{T}}B_i^{\mathrm{T}} + B_jY_i + Y_i^{\mathrm{T}}B_j^{\mathrm{T}} + 2Z - 4\beta X > 0 \\ X > 0, \quad Y \geq 0, \quad i < j, \quad \text{s.t.} \ \ h_i \cap h_j \neq \phi \\ \text{where} \ \ K_i = Y_iX^{-1}, \ X = P^{-1}, \ Z = XQX. \end{cases} \tag{A3}$$

Theorem 2.

Assume that the initial condition $x(o)$ is known, then[28−31]:

1) The constraint on control input $\|u(t)\|_2 < \varphi$ for $t \geq 0$ can be represented by

$$\begin{bmatrix} 1 & x(0)^{\mathrm{T}} \\ x(0) & X \end{bmatrix} \geq 0 \quad \text{(A4)}$$

$$\begin{bmatrix} X & Y_i^{\mathrm{T}} \\ Y_i^{\mathrm{T}} & \varphi^2 I \end{bmatrix} \geq 0. \quad \text{(A5)}$$

2) The constraint on output $\| y(t) \|_2 < \rho$ for $t \geq 0$ can be represented by

$$\begin{bmatrix} 1 & x(0)^{\mathrm{T}} \\ x(0) & X \end{bmatrix} \geq 0 \quad \text{(A6)}$$

$$\begin{bmatrix} X & XC_i^{\mathrm{T}} \\ XC_i^{\mathrm{T}} & \rho^2 . I \end{bmatrix} \geq 0 \quad \text{(A7)}$$

where

$$K_i = Y_i X^{-1}, \quad X = P^{-1}. \quad \text{(A8)}$$

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