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Anti-synchronization of Four-wing Chaotic Systems via Sliding Mode Control

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Abstract: Sliding mode control is an important method used in nonlinear control systems. In robust control systems, the sliding mode control is often adopted due to its inherent advantages of easy realization, fast response and good transient performance as well as its insensitivity to parameter uncertainties and disturbances. In this paper, we derive new results based on the sliding mode control for the anti-synchronization of identical Qi three-dimensional (3D) four-wing chaotic systems (2008) and identical Liu 3D four-wing chaotic systems (2009). The stability results for the anti-synchronization schemes derived in this paper using sliding mode control (SMC) are established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the SMC method is very effective and convenient to achieve global chaos anti-synchronization of the identical Qi four-wing chaotic systems and identical Liu four-wing chaotic systems. Numerical simulations are shown to illustrate and validate the synchronization schemes derived in this paper.

Keywords: Hybrid synchronization, chaos, sliding control, Qi four-wing chaotic system, Liu four-wing chaotic system.

1 Introduction

Chaotic systems are nonlinear dynamical systems which are highly sensitive to initial conditions. This sensitivity is popularly known as the butterfly effect[1]. In control theory, sliding mode control, or SMC, is a nonlinear control method that alters the dynamics of a nonlinear system by application of a discontinuous control signal that forces the system to "slide" along a cross-section of the system′s normal behaviour. In robust control systems, sliding mode control is often adopted due to its inherent advantages of easy realization, fast response and good transient performance as well as its insensitivity to parameter uncertainties and disturbances.

This paper has been organized as follows. In Section 2, we describe a survey of the previous work done in this area in chaos literature. In Section 3, we describe the problem statement and our methodology using sliding mode control. In Section 4, we discuss the global chaos anti-synchronization of identical Qi four-wing chaotic systems. In Section 5, we discuss the anti-synchronization of identical Liu four-wing chaotic systems. In Section 6, we summarize the main results obtained in this paper.

2 Literature survey

Since the seminal work by Pecora and Carroll[2], chaos synchronization problem has been studied extensively in [2–17]. Chaos theory has been successfully applied to a variety of fields, such as physical systems[3], chemical systems[4], ecological systems[5], secure communications[6−8], etc.

In the last two decades, various schemes have been successfully applied for chaos synchronization, such as polynomial chaos method[2], Ott Grebogi Yorke method[9], active control method[10−12], adaptive control method[13−15], time-delay feedback method[16,17], backstepping design method[18,19], sampled-data feedback synchronization method[20], etc.

In most of the chaos synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the master or drive system and another chaotic system is called the slave or response system, then the goal of the anti-synchronization is to use the output of the master system to control the slave system such that the states of the slave system have the same amplitude but opposite signs as the states of the master system asymptotically. In other words, the sums of the states of the master and slave systems are expected to converge to zero asymptotically, when anti-synchronization appears.

In this paper, we derive new results based on the sliding mode control[21−23] for the global chaos anti-synchronization of identical Qi three-dimensional (3D) four-wing chaotic systems[24] and identical Liu 3D four-wing chaotic systems[25]. Our stability results for anti-synchronization are established using Lyapunov stability theory[26].

In control theory, sliding mode control, or siliding mode control (SMC), is a nonlinear control method that alters the dynamics of a nonlinear system by application of a discontinuous control signal that forces the system to "slide" along a cross-section of the system′s normal behaviour. In robust control systems, sliding mode control is often adopted due to its inherent advantages of easy realization, fast response and good transient performance as well as its insensitivity to parameter uncertainties and disturbances.

3 Problem statement and our methodology using SMC

Consider the chaotic system described by

$$\dot{x} = Ax + f(x) \tag{1}$$

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where $x \in \mathbf{R}^n$ is the state of the system, A is the $n \times n$ matrix of the system parameters and f: $\mathbf{R}^n \to \mathbf{R}^n$ is the nonlinear part of the system.

We consider the system (1) as the master or drive system.

As the slave or response system, we consider the following chaotic system described by the dynamics

$$\dot{y} = Ay + f(y) + u \tag{2}$$

where $y \in \mathbf{R}^n$ is the state of the system and $u \in \mathbf{R}^m$ is the controller to be designed.

We define the anti-synchronization error as

$$e = y + x. \tag{3}$$

Then, the error dynamics is obtained as

$$\dot{e} = Ae + \eta(x, y) + u \tag{4}$$

where

$$\eta(x, y) = f(y) + f(x). \tag{5}$$

The objective of the global chaos synchronization problem is to find a controller u such that

$$\lim_{t\to\infty} \|e(t)\| = 0 \quad \text{for all } e(0) \in \mathbf{R}^n.$$

To solve this problem, we first define the control u as

$$u = -\eta(x, y) + Bv \tag{6}$$

where B is a constant gain vector selected such that (A, B) is controllable.

Substituting (6) into (4), the error dynamics simplifies to

$$\dot{e} = Ae + Bv \tag{7}$$

which is a linear time-invariant control system with single input v.

Thus, the original global chaos anti-synchronization problem can be replaced by an equivalent problem of stabilizing the zero solution $e = 0$ of the system (7) by a suitable choice of the sliding mode control.

In the sliding mode control, we define the variable

$$s(e) = Ce = c_1e_1 + c_2e_2 + \cdots + c_ne_n \tag{8}$$

where

$$C = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix}$$

is a constant vector to be determined.

In the SMC, we constrain the motion of the system (7) to the sliding manifold defined by

$$S = \{x \in \mathbf{R}^n | s(e) = 0\}$$

which is required to be invariant under the flow of the error dynamics (7).

When in sliding manifold S, the system (7) satisfies the following conditions:

$$s(e) = 0 \tag{9}$$

which is the defining equation for the manifold S and

$$\dot{s}(e) = 0 \tag{10}$$

which is the necessary condition for the state trajectory $e(t)$ of (7) to stay on the sliding manifold S.

Using (7) and (8), (10) can be rewritten as

$$\dot{s}(e) = C\,[Ae + Bv] = 0. \tag{11}$$

Solving (11) for v, we obtain the equivalent control law

$$v_{eq}(t) = -(CB)^{-1}CAe(t) \tag{12}$$

where C is chosen such that $CB \neq 0$.

Substituting (12) into the error dynamics (7), we obtain the closed-loop dynamics as

$$\dot{e} = [I - B(CB)^{-1}C]Ae. \tag{13}$$

The row vector C is selected such that the system matrix of the controlled dynamics

$$[I - B(CB)^{-1}C]A$$

is Hurwitz, i.e., all its eigenvalues have negative real parts. Then, the controlled system (13) is globally asymptotically stable.

To design the sliding mode controller for (7), we apply the constant plus proportional rate reaching law

$$\dot{s} = -q\,\text{sgn}(s) - ks \tag{14}$$

where $\text{sgn}(\cdot)$ denotes the sign function and the gains $q > 0, k > 0$ are determined such that the sliding condition is satisfied and sliding motion will occur.

From (11) and (14), we can obtain the control $v(t)$ as

$$v(t) = -(CB)^{-1}\left[C(kI + A)e + q\,\text{sgn}(s)\right] \tag{15}$$

which yields

$$v(t) = \begin{cases} -(CB)^{-1}\left[C(kI + A)e + q\right], & \text{if } s(e) > 0 \\ -(CB)^{-1}\left[C(kI + A)e - q\right], & \text{if } s(e) < 0. \end{cases} \tag{16}$$

Theorem 1. The master system (1) and the slave system (2) are globally and asymptotically anti-synchronized for all initial conditions $x(0), y(0) \in \mathbf{R}^n$ by the feedback control law

$$u(t) = -\eta(x, y) + Bv(t) \tag{17}$$

where $v(t)$ is defined by (15) and B is a column vector such that (A, B) is controllable. Also, the sliding mode gains k, q are positive.

Proof. First, we note that substituting (17) and (15) into the error dynamics (4), we obtain the closed-loop error dynamics as

$$\dot{e} = Ae - B(CB)^{-1}[C(kI + A)e + q\,\text{sgn}(s)]. \tag{18}$$

To prove that the error dynamics (18) is globally asymptotically stable, we consider the candidate Lyapunov function defined by the equation

$$V(e) = \frac{1}{2}s^2(e) \tag{19}$$

which is a positive definite function on $\mathbf{R}^n$.

Differentiating V along the trajectories of (18) or the equivalent dynamics (14), we get

$$\dot{V}(e) = s(e)\dot{s}(e) = -ks^2 - q\,\text{sgn}(s)s \tag{20}$$

which is a negative definite function on $\mathbf{R}^n$.

This calculation shows that V is a globally defined, positive definite, Lyapunov function for the error dynamics (18), which has a globally defined, negative definite time derivative $\dot{V}$.

Thus, by Lyapunov stability theory[26], it is immediate that the error dynamics (18) is globally asymptotically stable for all initial conditions $e(0) \in \mathbf{R}^n$.

Hence, it follows that the master system (1) and the slave system (2) are globally and asymptotically anti-synchronized for all initial conditions $x(0), y(0) \in \mathbf{R}^n$. □

4 Anti-synchronization of identical Qi four-wing chaotic systems via SMC

In this section, we apply the sliding mode control results derived in Section 3 for the global chaos anti-synchronization of identical Qi four-wing chaotic systems[24].

Thus, the master system is described by the Qi dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + \varepsilon x_2 x_3 \\ \dot{x}_2 &= cx_1 + dx_2 - x_1 x_3 \\ \dot{x}_3 &= -bx_3 + x_1 x_2 \end{aligned} \tag{21}$$

where x_1, x_2, x_3 are state variables, a, b, d are all real positive, constant parameters and c, ε are real constant parameters of the system.

The Qi system is chaotic when the parameter values are $a = 14, b = 43, c = -1, d = 16$ and $\varepsilon = 4$.

Fig. 1 illustrates the state orbits of the Qi four-wing chaotic system (21).

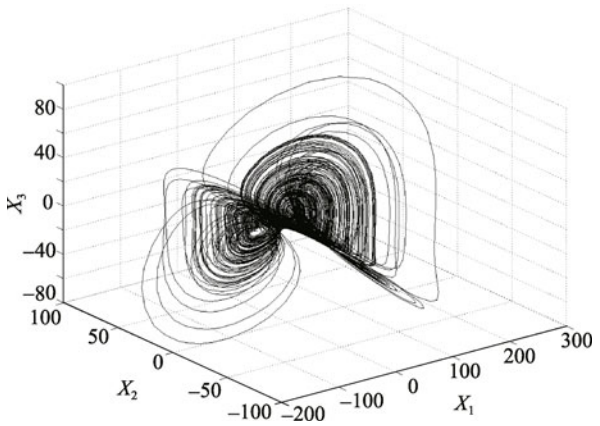

Fig. 1 State orbits of the Qi four-wing chaotic system

The slave system is described by the controlled Qi dynamics

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + \varepsilon y_2 y_3 + u_1 \\ \dot{y}_2 &= cy_1 + dy_2 - y_1 y_3 + u_2 \\ \dot{y}_3 &= -by_3 + y_1 y_2 + u_3 \end{aligned} \tag{22}$$

where y_1, y_2, y_3 are state variables and u_1, u_2, u_3 are the controllers to be designed.

The anti-synchronization error is defined by

$$e_i = y_i + x_i, \ (i = 1, 2, 3). \tag{23}$$

The error dynamics is easily obtained as

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + \varepsilon(y_2 y_3 + x_2 x_3) + u_1 \\ \dot{e}_2 &= ce_1 + de_2 - y_1 y_3 - x_1 x_3 + u_2 \\ \dot{e}_3 &= -be_3 + y_1 y_2 + x_1 x_2 + u_3. \end{aligned} \tag{24}$$

We write the error dynamics (24) in matrix notation as

$$\dot{e} = Ae + \eta(x, y) + u \tag{25}$$

where

$$A = \begin{bmatrix} -a & a & 0 \\ c & d & 0 \\ 0 & 0 & -b \end{bmatrix}, \tag{26}$$

$$\eta(x, y) = \begin{bmatrix} \varepsilon(y_2 y_3 + x_2 x_3) \\ -y_1 y_3 - x_1 x_3 \\ y_1 y_2 + x_1 x_2 \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}. \tag{27}$$

The sliding mode controller design is carried out as detailed in Section 3.

First, we set u as

$$u = -\eta(x, y) + Bv \tag{28}$$

where B is chosen such that (A, B) is controllable.

We take B as

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \tag{29}$$

In the chaotic case, the parameter values are $a = 14, b = 43, c = -1, d = 16$ and $\varepsilon = 4$.

The sliding mode variable is selected as

$$s = Ce = [2 \quad 8 \quad 1]e \tag{30}$$

which makes the sliding mode state equation asymptotically stable.

We choose the sliding mode gains as $k = 4$ and $q = 0.2$.

We note that a large value of k can cause chattering and an appropriate value of q is chosen to speed up the time taken to reach the sliding manifold as well as to reduce the system chattering.

From (15), we can obtain $v(t)$ as

$$v = 2.546e_1 - 17.091e_2 + 3.546e_3 - 0.018\text{sgn}(s). \tag{31}$$

Thus, the required sliding mode controller is obtained as

$$u = -\eta(x, y) + Bv \tag{32}$$

where $\eta(x, y)$, B and $v(t)$ are defined as in (27), (29), and (31).

By Theorem 1, we obtain the following result.

Theorem 2. The identical Qi four-wing systems (21) and (22) are globally and asymptotically anti-synchronized for all initial conditions with the sliding mode controller u defined by (31).

4.1 Numerical results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the Qi four-wing chaotic systems (21) and (22) with the sliding mode controller given by (32) using Matlab.

The initial values of the master system (21) are taken as

$$x_1(0) = 5, \quad x_2(0) = 12, \quad x_3(0) = 20$$

and the initial values of the slave system (22) are taken as

$$y_1(0) = 16, \quad y_2(0) = 24, \quad y_3(0) = 7.$$

Fig. 2 illustrates the anti-synchronization of the identical Qi four-wing chaotic systems (21) and (22).

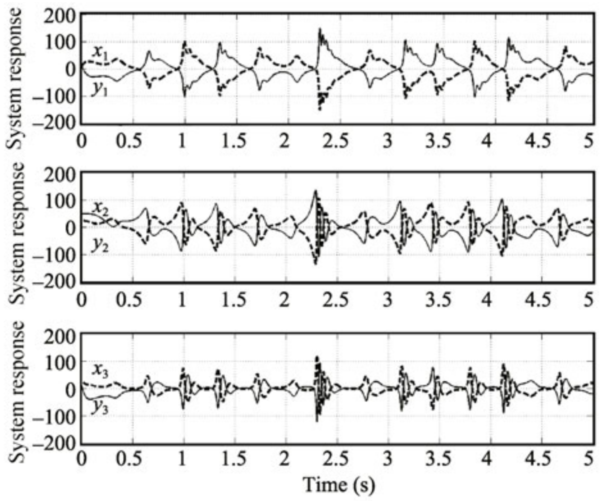

Fig. 2 Anti-synchronization of identical Qi four-wing systems

5 Anti-synchronization of identical Liu four-wing chaotic systems via SMC

In this section, we apply the sliding mode control results derived in Section 3 for the global chaos anti-synchronization of identical Liu four-wing chaotic systems[25].

Thus, the master system is described by the Liu dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_2 x_3^2 \\ \dot{x}_2 &= b(x_1 + x_2) - x_1 x_3^2 \\ \dot{x}_3 &= -cx_3 + \varepsilon x_2 + x_1 x_2 x_3 \end{aligned} \tag{33}$$

where x_1, x_2, and x_3 are state variables, are all real positive, constant parameters of the system.

The Liu system is chaotic when the parameter values are

$$a = 50, \quad b = 13, \quad c = 13, \quad \varepsilon = 6.$$

The slave system is described by the controlled Liu dynamics

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + y_2 y_3^2 + u_1 \\ \dot{y}_2 &= b(y_1 + y_2) - y_1 y_3^2 + u_2 \\ \dot{y}_3 &= -cy_3 + \varepsilon y_2 + y_1 y_2 y_3 + u_3 \end{aligned} \tag{34}$$

where y_1, y_2, and y_3 are state variables and u_1, u_2, and u_3 are the controllers to be designed.

Fig. 3 illustrates the state orbits of the Liu four-wing chaotic system (21).

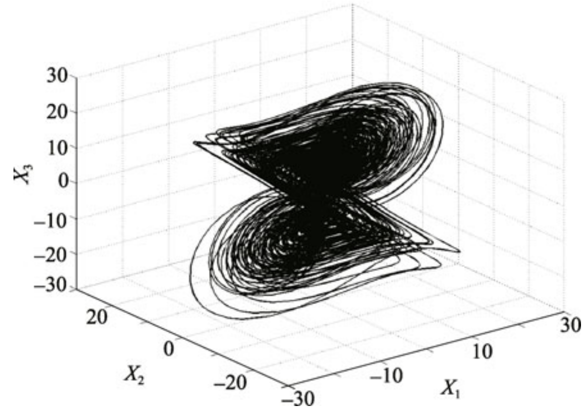

Fig. 3 State orbits of the Liu four-wing chaotic system

The anti-synchronization error is defined by

$$e_i = y_i + x_i, \ (i = 1, 2, 3). \tag{35}$$

The error dynamics is easily obtained as

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + y_2 y_3^2 + x_2 x_3^2 + u_1 \\ \dot{e}_2 &= b(e_1 + e_2) - y_1 y_3^2 - x_1 x_3^2 + u_2 \\ \dot{e}_3 &= -ce_3 + \varepsilon e_2 + y_1 y_2 y_3 + x_1 x_2 x_3 + u_3. \end{aligned} \tag{36}$$

We write the error dynamics (24) in matrix notation as

$$\dot{e} = Ae + \eta(x, y) + u \tag{37}$$

where

$$A = \begin{bmatrix} -a & a & 0 \\ b & b & 0 \\ 0 & 0 & -c \end{bmatrix} \tag{38}$$

$$A = \begin{bmatrix} y_2 y_3^2 + x_2 x_3^2 \\ -y_1 y_3^2 - x_1 x_3^2 \\ y_1 y_2 y_3 + x_1 x_2 x_3 \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}. \tag{39}$$

The sliding mode controller design is carried out as detailed in Section 3.

First, we set u as

$$u = -\eta(x, y) + Bv \tag{40}$$

where B is chosen such that (A, B) is controllable.

We take B as

$$B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}. \tag{41}$$

In the chaotic case, the parameter values are

$$a = 50, \quad b = 13, \quad c = 13, \quad \varepsilon = 6.$$

The sliding mode variable is selected as

$$s = Ce[2 \quad 8 \quad 1]e \tag{42}$$

which makes the sliding mode state equation asymptotically stable.

We choose the sliding mode gains as $k = 4$ and $q = 0.2$.

We note that a large value of k can cause chattering and an appropriate value of q is chosen to speed up the time taken to reach the sliding manifold as well as to reduce the system chattering.

From (15), we can obtain $v(t)$ as

$$v = -1.091e_1 - 21.455e_2 + 0.818e_3 - 0.018\mathrm{sgn}(s). \quad (43)$$

Thus, the required sliding mode controller is obtained as

$$u = -\eta(x, y) + Bv \quad (44)$$

where $\eta(x, y), B$ and $v(t)$ are defined as in (27), (29) and (31).

By Theorem 1, we obtain the following result.

Theorem 3. The identical Liu four-wing systems (33) and (34) are globally and asymptotically anti-synchronized for all initial conditions with the sliding mode controller defined by (44). □

5.1 Numerical results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the Liu four-wing chaotic systems (33) and (34) with the sliding mode controller given by (44) using Matlab.

The initial values of the master system (33) are taken as

$$x_1(0) = 12, \quad x_2(0) = 22, \quad x_3(0) = 10$$

and the initial values of the slave system (34) are taken as

$$y_1(0) = 4, \quad y_2(0) = 20, \quad y_3(0) = 14.$$

Fig. 4 illustrates the anti-synchronization of the identical Liu four-wing chaotic systems (33) and (34).

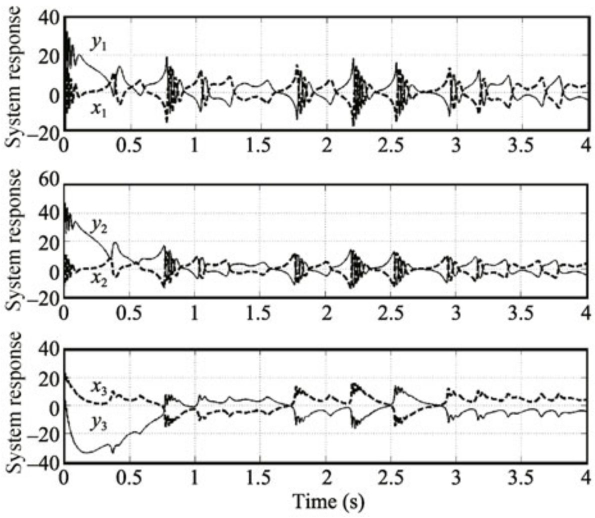

Fig. 4 Anti-synchronization of identical Liu four-wing systems

6 Conclusions

In this paper, we have deployed sliding mode control (SMC) to achieve global chaos anti-synchronization for the Qi four-wing chaotic systems[24] and for the Liu four-wing chaotic systems[25]. Our synchronization results have been proved using the Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the sliding mode control is very effective and convenient to achieve global chaos anti-synchronization for identical Qi four-wing chaotic systems and Liu four-wing chaotic systems. Numerical simulations are also shown to validate synchronization results derived in this paper.

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