International Journal of Automation and Computing 7(4), November 2010, 455-462
DOI: 10.1007/s11633-010-0527-3

Delay and Its Time-derivative Dependent Robust Stability of Uncertain Neutral Systems with Saturating Actuators

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Abstract: This note concerns the problem of the robust stability of uncertain neutral systems with time-varying delay and saturating actuators. The system considered is continuous in time with norm bounded parametric uncertainties. By incorporating the free weighing matrix approach developed recently, some new delay-dependent stability conditions in terms of linear matrix inequalities (LMIs) with some tuning parameters are obtained. An estimate of the domain of attraction of the closed-loop system under a priori designed controller is proposed. The approach is based on a polytopic description of the actuator saturation nonlinearities and the Lyapunov-Krasovskii method. Numerical examples are used to demonstrate the effectiveness of the proposed design method.

Keywords: Actuator saturation, neutral systems, uncertainty, delay-dependent condition, linear matrix inequality (LMI), time varying delay systems.

1 Introduction

The problem of delayed systems has been investigated over the years because the phenomena of time-delay are very often encountered in different technical systems, such as electric, pneumatic, and hydraulic networks, chemical processes, long transmission lines, etc. Time delay is often a source of instability and oscillation in practical systems. The robust stability of uncertain systems with time delays has received considerable attention. Existing criteria for asymptotic stability of time-delay systems can be classified into two types: delay-independent stability[1, 2] and delay-dependent stability[3−10]. The former does not include any information on the size of delay while the latter employs such information. It is well known that delay-independent criteria tend to be conservative, especially when the size of a delay is small. A major problem in the control of linear dynamical systems with time delay is that the actuator saturations are unavoidable. The actuator saturation not only deteriorates the control system performance, but also leads to undesirable stability effects. The stability analysis and stabilization of time delay systems with saturating actuator have been widely investigated by many researchers. References [11–13] have treated this problem and obtained saturating control laws that govern the system stability when the initial state belongs to an estimated domain. By using the Lyapunov method, a number of delay-dependent robust stabilization techniques for a class of uncertain state-delayed systems have been investigated via predicted-based transformation[8]. Fridman and Shaked[4] combined Park′s and Moon′s inequalities with a descriptor model transform and found rather efficient criteria for systems with polytopic-type uncertainties. Recently, a free weighting matrix approach to overcoming the conservativeness of methods involving a fixed model transformation[6, 14] is expected to be able to further improve the performance when applied to delay-dependent stability with some tuning parameters. This has motivated the work in this paper. In the more general case of neutral-type systems where the delay appears in the state derivative and in the state, several sufficient conditions have been obtained for delay-independent[15] and the delay-dependent[16−23] cases. Note that unlike retarded-type systems, neutral systems may be destabilized by small changes in delays[24]. This work aims to develop some delay-dependent methods for neutral systems with a time-varying delay, actuators constraints, and norm-bounded parametric uncertainties via linear memoryless state feedback control law. The control law serves to guarantee the local stability of the closed loop system when the initial states are taken in a predetermined region of attraction. In the following, a linear matrix inequality (LMI) optimization approach will be proposed to design the state feedback gain for maximizing this estimate of the domain of attraction. A less conservative estimate of the region of attraction will be derived based on the Lyapunov-Krasovskii functional approach. The conditions are given in terms of LMIs. Note that the LMIs approach has the advantage that it can be solved numerically very efficiently using the interior point algorithm developed in [25, 26] and using Matlab LMI toolbox[27]. Three numerical examples are given to illustrate that the results are less conservative than previous work.

2 Problem formulation and definitions

Consider the following uncertain neutral system with time-varying delay:

$$\dot{x}(t) - C\dot{x}(t-\tau(t)) = (A_0 + \Delta A_0(t))x(t) + (A_1 + \Delta A_1(t)) \times x(t-h(t)) + (B + \Delta B(t))\mathrm{sat}(u(t)),\quad t > 0 \tag{1}$$

Manuscript received May 31, 2009; revised August 19, 2009

where $x(t) \in \mathbf{R}^n$ is the state vector; $u(t) \in \mathbf{R}^m$ is the control input; C, A_0, A_1, and B are known real constant matrices. The delays τ and h are assumed to be some unknown functions of time and are continuously differentiable, with their respective rates of change bounded as follows:

$$0 \leqslant h(t) \leqslant h_m, \quad 0 \leqslant \tau(t) < \infty \tag{2}$$

and

$$0 \leqslant \dot{h}(t) \leqslant d_1 < 1, \quad 0 \leqslant \dot{\tau}(t) \leqslant d_2 < 1 \tag{3}$$

where h_m, d_1, and d_2 are given positive constants. Note that condition (3) should be satisfied so as to ensure that system (1) admits a physical meaningful solution compatible with the causality principle (see [28] for more discussions on varying delays and derivative bounds).

The initial condition of system (1) is given by

$$x(\theta) = \phi(\theta), \;\; \theta \in [-\bar{h}, 0]$$

where $\bar{h} = \max\{\tau(t), h(t)\}$, $\forall t \geqslant 0$, and $\phi(\cdot)$ is a differentiable vector valued initial function.

Define the operator $\Delta : C^1[-\bar{h}, 0] \rightarrow \mathbf{R}^n$ as $\Delta(x_t) = x(t) - Cx(t-\tau)$.

Assumption 1. All the eigenvalues of matrix C are inside the unit circle.

In this paper, we assume that the uncertainties can be described as follows:

$$[\Delta A_0(t) \;\; \Delta A_1(t) \;\; \Delta B(t)] = DF(t)[E_0 \;\; E_1 \;\; E_2] \tag{4}$$

where D, E_0, E_1, and E_2 are known constant real matrices of appropriate dimensions, and $F(t)$ denotes time-varying parameter uncertainties and is assumed to be of block diagonal form $F(t) = \mathrm{diag}\{F_1(t), \cdots, F_r(t)\}$, where $F_i(t) \in \mathbf{R}^{p_i \times q_i}$; $i = 1, \cdots, r$ are unknown real time-varying matrices satisfying $F_i^{\mathrm{T}}(t)F_i(t) \leqslant I$, $\forall t \geqslant 0$.

The saturation function is defined by

$$\mathrm{sat}(u(t)) = [\mathrm{sat}(u_1(t)), \mathrm{sat}(u_2(t)), \cdots, \mathrm{sat}(u_m(t))]^{\mathrm{T}} \tag{5}$$

and

$$\mathrm{sat}(u_i(t)) = \begin{cases} \overline{u}_i, & \text{if} \quad u_i > \overline{u}_i \\ u_i, & \text{if} \quad -\overline{u}_i \leqslant u_i \leqslant \overline{u}_i \\ -\overline{u}_i, & \text{if} \quad u_i < -\overline{u}_i. \end{cases}$$

Lemma 1[29]. Let D, E, and $F(t)$ be real matrices of appropriate dimensions with $F = \mathrm{diag}\{F_1, \cdots, F_r\}$, $F_i^{\mathrm{T}} F_i \leqslant I$, $i = 1, \cdots, r$. Then, for any real matrix $\Lambda = \mathrm{diag}\{\mu_1 I, \cdots, \mu_r I\} > 0$, the following inequality is true:

$$DF(t)E + E^{\mathrm{T}}F(t)^{\mathrm{T}}D^{\mathrm{T}} \leqslant D\Lambda D^{\mathrm{T}} + E^{\mathrm{T}}\Lambda^{-1}E.$$

In this paper, we consider the stabilization of system (1) using a linear state feedback

$$u(t) = Kx(t). \tag{6}$$

For an initial condition $x_0 = \phi \in C^1[-\bar{h}; 0]$, denote the state trajectory of system (1) by $x(t, \phi)$. Suppose that the solution $x(t) = 0$ is asymptotically stable for all delays satisfying (2). Then, the domain of attraction of the origin is

$$\Psi = \{\phi \in C^1[-\bar{h}; 0] : \lim_{t\rightarrow\infty} x(t, \phi) = 0\}.$$

Moreover, we are interested in obtaining an estimate $\Xi_\delta \subset \Psi$ of the domain of attraction, where

$$\Xi_\delta = \{\phi \in C^1[-\bar{h}; 0] : \max_{[-\bar{h};0]} |\phi| \leqslant \delta\}$$

and where $\delta > 0$ is a scalar to be maximized in the sequel.

Define the polyhedron

$$D(K, \bar{u}) = \{x \in \mathbf{R}^n; |k_i x| \leqslant \bar{u}_i, i = 1, \cdots, m\}$$

where k_i denotes the i-th row of K. We exploit the idea of [11] in the development of the results of this paper. Denote the set of all diagonal matrices in $\mathbf{R}^{m\times m}$ with diagonal elements that are 1 or 0 by N. Then, there are 2^m elements D_i in N, where N denotes the set of diagonal matrices with diagonal elements are 0 or 1, and for every $i = 1, \cdots, 2^m$, $D_i^- = I - D_i$ is also an element in N.

Lemma 2[11]. Given K and H in $\mathbf{R}^{m\times n}$, we have

$$\mathrm{sat}(Kx(t)) \in Co\{D_iKx + D_i^- Hx, i = 1, \cdots, 2^m\}$$

for all $x \in \mathbf{R}^n$ that satisfy $|h_i x| \leqslant \bar{u}_i$, $i = 1, \cdots, m$.

Therefore, for $x \in S_c$, any compact set of $\mathbf{R}^n$, let H be in $\mathbf{R}^{m\times n}$ such that $|h_i x| \leqslant \bar{u}_i$. Then, the motion of the system (1)–(4) can be described by the following system:

$$\dot{x}(t) - C\dot{x}(t-\tau(t)) = \sum_{j=1}^{2^m} \lambda_j \widehat{A}_j x(t) + \overline{A}_1 x(t-h(t)) \tag{7}$$

where $\widehat{A}_j = \overline{B}(D_jK + D_j^- H) + \overline{A}_0$, $\sum_{j=1}^{2^m} \lambda_j = 1$, $\lambda_j \geqslant 0$ $\overline{A}_0 = A_0 + DF(t)E_0$, $\overline{A}_1 = A_1 + DF(t)E_1$, and $\overline{B} = B + DF(t)E_2$.

Choose a Lyapunov functional candidate to be

$$V(t) = x^{\mathrm{T}}(t)Px(t) + \int_{t-h(t)}^{t} x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s + \\ \int_{-h_m}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s)R\dot{x}(s)\mathrm{d}s\mathrm{d}\theta + \int_{t-\tau(t)}^{t} \dot{x}^{\mathrm{T}}(s)W\dot{x}(s)\mathrm{d}s \tag{8}$$

where $P = P^{\mathrm{T}} > 0$, $Q = Q^{\mathrm{T}} > 0$, $R = R^{\mathrm{T}} > 0$, $W = W^{\mathrm{T}} > 0$.

A convenient choice of the set S_c can be defined from a symmetric positive definite matrix P as

$$D_e = \{x(t) \in \mathbf{R}^n; \;\; x^{\mathrm{T}}(t)Px(t) \leqslant \beta^{-1}\} \tag{9}$$

where β is a positive scalar.

3 Main results

In this section, we will give some sufficient conditions for (1)–(4) to be robustly stable.

Lemma 3. Under Assumption 1, the system described by (1)–(4) is robustly stable if there exist $P = P^{\mathrm{T}} > 0$, $Q = Q^{\mathrm{T}} > 0$, $R = R^{\mathrm{T}} > 0$, $W = W^{\mathrm{T}} > 0$, and appropriately dimensioned matrices Y_1, Y_2, T_1, and T_2 such that the following LMIs hold:

$$\Gamma_j = \begin{pmatrix} \Gamma_{11(j)} & \Gamma_{21(j)}^{\rm T} & \Gamma_{31}^{\rm T} & h_mY_1 & T_1C \\ \Gamma_{21(j)} & \Gamma_{22} & \Gamma_{32}^{\rm T} & h_mY_2 & T_2C \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & 0 & 0 \\ h_mY_1^{\rm T} & h_mY_2^{\rm T} & 0 & -h_mR & 0 \\ C^{\rm T}T_1^{\rm T} & C^{\rm T}T_2^{\rm T} & 0 & 0 & -(1-d_2)W \end{pmatrix} < 0$$

$$j = 1, \cdots, 2^m \qquad (10)$$

where

$$\begin{cases} \Gamma_{11(j)} = T_1\widehat{A}_j + \widehat{A}_j^{\rm T}T_1^{\rm T} + Y_1 + Y_1^{\rm T} + Q \\ \Gamma_{21(j)} = P + T_2\widehat{A}_j + Y_2 - T_1^{\rm T} \\ \Gamma_{22} = h_mR + W - T_2 - T_2^{\rm T} \\ \Gamma_{31} = \overline{A}_1^{\rm T}T_1^{\rm T} - Y_1^{\rm T} \\ \Gamma_{32} = \overline{A}_1^{\rm T}T_2^{\rm T} - Y_2^{\rm T} \\ \Gamma_{33} = -(1-d_1)Q. \end{cases}$$

Proof. Calculating the derivative of $V(t)$ along the solution of system (1) and using (2) yields

$$\dot{V}(t) \leqslant 2x^{\rm T}(t)P\dot{x}(t) + x^{\rm T}(t)Qx(t) - (1-d_1)x^{\rm T}(t-h(t))Q\times$$
$$x(t-h(t)) + h_m\dot{x}^{\rm T}(t)R\dot{x}(t) - \int_{t-h(t)}^{t}\dot{x}^{\rm T}(s)R\dot{x}(s){\rm d}s +$$
$$\dot{x}^{\rm T}(t)W\dot{x}(t) - (1-d_2)\dot{x}^{\rm T}(1-\tau(t))W\dot{x}(1-\tau(t)). \qquad (11)$$

Using the free weighting matrix approach introduced in [6], for appropriately matrices Y_1, Y_2, T_1, and T_2, we have

$$2[x^{\rm T}(t)T_1 + \dot{x}^{\rm T}(t)T_2] \times [-\dot{x}(t) + C\dot{x}(1-\tau(t)) + \widehat{A}_jx(t) +$$
$$\overline{A}_1x(t-h(t))] = 0, \qquad j = 1, \cdots, 2^m$$
$$2[x^{\rm T}(t)Y_1 + \dot{x}^{\rm T}(t)Y_2] \times [x(t) - x(t-h(t)) -$$
$$\int_{t-h(t)}^{t}\dot{x}(s){\rm d}s] = 0.$$

For any positive semi-definite matrix

$$Z = \begin{pmatrix} Z_{11} & Z_{21}^{\rm T} & Z_{31}^{\rm T} \\ Z_{21} & Z_{22} & Z_{32}^{\rm T} \\ Z_{31} & Z_{32} & Z_{33} \end{pmatrix} \geqslant 0$$

and

$$\eta(t) = \begin{pmatrix} x(t) \\ \dot{x}(t) \\ x(t-h(t)) \end{pmatrix}$$

we also have

$$h_m\eta^{\rm T}(t)Z\eta(t) - \int_{t-h(t)}^{t}\eta^{\rm T}(t)Z\eta(t){\rm d}s \geqslant 0.$$

Then, adding those terms to the right-hand side of (11), letting

$$\Omega(t,s) = \begin{pmatrix} \xi(t,s) \\ \dot{x}(t-\tau(t)) \end{pmatrix}, Z = \begin{pmatrix} Y_1 \\ Y_2 \\ 0 \end{pmatrix} R^{-1} \begin{pmatrix} Y_1 \\ Y_2 \\ 0 \end{pmatrix}^{\rm T}$$

and using the Schur complement we obtain $\dot{V}(t) \leqslant \sum_{j=1}^{2^m}\lambda_j\Omega^{\rm T}(t,s)\Gamma_j\Omega(t,s)$.

Then, there exists π such that $\dot{V}(t) \leqslant -\pi\|x(t)\|^2$, which ensures the asymptotic stability of system (1)–(4) according to [30]. □

Theorem 1. For given $\varepsilon_1, \varepsilon_2 \in \mathbf{R}$, if there exist $\overline{Q} = \overline{Q}^{\rm T} > 0, \overline{R} = \overline{R}^{\rm T} > 0, \overline{W} = \overline{W}^{\rm T} > 0$, $X_1 = X_1^{\rm T} > 0$, $X_2, X_3 \in \mathbf{R}^{n\times n}$, $U, G \in \mathbf{R}^{m\times n}$, $\Lambda = {\rm diag}\{\mu_1I, \cdots, \mu_rI\} > 0$, and positive scalars β, δ, satisfying the following linear matrix inequalities[1]:

$$\begin{pmatrix} \beta & * \\ g_i^{\rm T} & \bar{u}_i^2X_1 \end{pmatrix} \geqslant 0, \quad i = 1, \cdots, m \qquad (13)$$

where

$$\begin{cases} \Sigma_{11} = X_2 + X_2^{\rm T} + \varepsilon_1(X_1A_1^{\rm T} + A_1X_1) \\ \Sigma_{21(j)} = X_3^{\rm T} - X_2 + (A_0 + \varepsilon_2A_1)X_1 + \\ \qquad B(D_jU + D_j^-G) \\ \Sigma_{22} = -X_3^{\rm T} - X_3 + D\Lambda D^{\rm T} \end{cases}$$

and g_i denotes the i-th row of G, then control law (6) with $K = UX_1^{-1}$ stabilizes system (1)–(4) for every initial condition in Ξ_δ with

$$\begin{pmatrix} \Sigma_{11} & * & * & * & * & * & * & * & * \\ \Sigma_{21(j)} & \Sigma_{22} & * & * & * & * & * & * & * \\ -\varepsilon_1\overline{Q}A_1^{\rm T} & (1-\varepsilon_2)\overline{Q}A_1^{\rm T} & -(1-d_1)\overline{Q} & * & * & * & * & * & * \\ h_m\varepsilon_1\overline{R}A_1^{\rm T} & h_m\varepsilon_2\overline{R}A_1^{\rm T} & 0 & -h_m\overline{R} & * & * & * & * & * \\ 0 & \overline{W}C^{\rm T} & 0 & 0 & -(1-d_2)\overline{W} & * & * & * & * \\ h_mX_2 & h_mX_3 & 0 & 0 & 0 & -h_m\overline{R} & * & * & * \\ X_2 & X_3 & 0 & 0 & 0 & 0 & -\overline{W} & * & * \\ X_1 & 0 & 0 & 0 & 0 & 0 & 0 & -\overline{Q} & * \\ E_0X_1 + E_2(D_jU + D_j^-G) & 0 & E_1\overline{Q} & 0 & 0 & 0 & 0 & 0 & -\Lambda \end{pmatrix} < 0$$

$$j = 1, \cdots, 2^m \qquad (12)$$

[1]The symbol $*$ stands for symmetric block in matrix inequalities.

$$\delta^2 \max\{\lambda_{\max}(X_1^{-1}) + 2\frac{h_m}{(1-d_1)}\lambda_{\max}(\overline{Q}^{-1}); \\ 2h_m^2\lambda_{\max}(\overline{Q}^{-1}) + \frac{1}{(1-d_2)}\lambda_{\max}(\overline{W}^{-1}) + \\ h_m\lambda_{\max}(\overline{R}^{-1})\} \leqslant \beta^{-1}. \tag{14}$$

Proof. From the requirement that $P = P^{\rm T} > 0$, and the fact that in (10), $(-T_2 - T_2^{\rm T})$ must be negative definite, it follows that $\widetilde{P}$ is non-singular with

$$\widetilde{P}^{-1} = X = \begin{pmatrix} P & 0 \\ T_1^{\rm T} & T_2^{\rm T} \end{pmatrix}^{-1} = \begin{pmatrix} X_1 & 0 \\ X_2 & X_3 \end{pmatrix}.$$

Then, multiply both sides of (10) by $\mathrm{diag}\{X^{\rm T}, I, I, I\}$ and $\mathrm{diag}\{X, I, I, I\}$, and introduce some changes of variables such that

$$X_1 = P^{-1},\ \overline{Q} = Q^{-1},\ \overline{R} = R^{-1},\ \overline{W} = W^{-1},\ U = KX_1,$$
$$G = HX_1,\ \begin{pmatrix} N_1 \\ N_2 \end{pmatrix} = \begin{pmatrix} X_1Y_1 + X_2^{\rm T}Y_2 \\ X_3^{\rm T}Y_2 \end{pmatrix} X_1.$$

By the Schur complement[25], LMI (10) implies (15) with $\Pi_{21(j)} = X_3^{\rm T} - X_2 + N_2 + \overline{A}_0X_1 + \overline{B}(D_jU + D_j^- G)$. The main difficulty in the application of condition (15) is the presence of some nonlinearities such as X_1^{-1}, N_1, and N_2. Unfortunately, the condition cannot be directly solved and there is the need to tune the variables. To overcome this, we choose

$$N_1 = \varepsilon_1 A_1 X_1,\ \ N_2 = \varepsilon_2 A_1 X_1 \tag{16}$$

where ε_1 and ε_2 are some decision variables, and use the simple procedure presented in Remark 3 in this section. Using (16), we obtain the following inequality:

$$M_j + \overline{D}F(t)\overline{E} + \overline{E}^{\rm T}F^{\rm T}(t)\overline{D}^{\rm T} < 0, \quad j = 1, \cdots, 2^m \tag{17}$$

where M_j is shown at the bottom of this page, $\bar{D}$ and $\bar{E}$ are as follows:

$$\overline{D} = \begin{pmatrix} 0 & D^{\rm T} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^{\rm T}$$

$$\overline{E} = \begin{pmatrix} E_0X_1 + \overline{Z} & 0 & E_1\overline{Q} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

where $\overline{Z} = E_2(D_jU + \bar{D}_jG)$.

According to Lemma 1, (17) holds if there exists $\Lambda = \mathrm{diag}\{\mu_1 I, \cdots, \mu_r I\} > 0$ such that

$$M_j + \overline{D}\Lambda\overline{D}^{\rm T} + \overline{E}^{\rm T}\Lambda^{-1}\overline{E} < 0. \tag{18}$$

Thus, by the Schur complement, (18) is equivalent to (12) of Theorem 1. Moreover, the satisfaction of LMI (13) guarantees that $|h_ix| \leqslant \bar{u}_i$, $\forall x \in D_e$, $i = 1, \cdots, m$. This can be proven in the same manner as in [11–13].

Furthermore, following [22], the Lyapunov functional defined in (8) can be shown to satisfy

$$\pi_1\|D\phi\|^2 \leqslant V(\phi) \leqslant \pi_2 \max_{[-\bar{h},0]} |\phi|^2$$

with $\pi_1 = \lambda_{\min}(X_1^{-1})$ and

$$\pi_2 = \max\{\lambda_{\max}(X_1^{-1}) + 2\frac{h_m}{(1-d_1)}\lambda_{\max}(\overline{Q}^{-1}); \\ 2h_m^2\lambda_{\max}(\overline{Q}^{-1}) + \frac{1}{(1-d_2)}\lambda_{\max}(\overline{W}^{-1}) + h_m\lambda_{\max}(\overline{R}^{-1})\}.$$

From $\dot{V} < 0$, it follows that $V(t) < V(0)$ and therefore

$$x^{\rm T}(t)X_1^{-1}x(t) \leqslant V(t) < V(0) \leqslant \max_{\theta\in[-\bar{h},0]} |\phi(\theta)|^2\pi_2 \leqslant \beta^{-1}.$$

$$\begin{pmatrix} X_2 + X_2^{\rm T} + N_1 + N_1^{\rm T} & * & * & * & * & * & * & * \\ \Pi_{21(j)} & -X_3 - X_3^{\rm T} & * & * & * & * & * & * \\ -X_1^{-1}N_1^{\rm T} & \overline{A}_1^{\rm T} - X_1^{-1}N_2^{\rm T} & -(1-d_1)\overline{Q}^{-1} & * & * & * & * & * \\ h_mX_1^{-1}N_1^{\rm T} & h_mX_1^{-1}N_2^{\rm T} & 0 & -h_m\overline{R}^{-1} & * & * & * & * \\ 0 & \overline{W}C^{\rm T} & 0 & 0 & -(1-d_2)\overline{W} & * & * & * \\ h_mX_2 & h_mX_3 & 0 & 0 & 0 & -h_m\overline{R} & * & * \\ X_2 & X_3 & 0 & 0 & 0 & 0 & -\overline{W} & * \\ X_1 & 0 & 0 & 0 & 0 & 0 & 0 & -\overline{Q} \end{pmatrix} < 0$$

$$j = 1, \cdots, 2^m \tag{15}$$

$$M_j = \begin{pmatrix} \Sigma_{11} & * & * & * & * & * & * & * \\ \Sigma_{21(j)} & \Sigma_{22} & * & * & * & * & * & * \\ -\varepsilon_1\overline{Q}A_1^{\rm T} & (1-\varepsilon_2)\overline{Q}A_1^{\rm T} & -(1-d_1)\overline{Q} & * & * & * & * & * \\ h_m\varepsilon_1\overline{R}A_1^{\rm T} & h_m\varepsilon_2\overline{R}A_1^{\rm T} & 0 & -h_m\overline{R} & * & * & * & * \\ 0 & \overline{W}C^{\rm T} & 0 & 0 & -(1-d_2)\overline{W} & * & * & * \\ h_mX_2 & h_mX_3 & 0 & 0 & 0 & -h_m\overline{R} & * & * \\ X_2 & X_3 & 0 & 0 & 0 & 0 & -\overline{W} & * \\ X_1 & 0 & 0 & 0 & 0 & 0 & 0 & -\overline{Q} \end{pmatrix}$$

Inequality (14) guarantees that for all initial functions $\phi \in \Xi_\delta$, the trajectories of $x(t)$ remain within D_e and $\dot{V} < 0$ along the trajectories of the closed loop system (7) which implies that $\lim_{t\to\infty} x(t) = 0$. □

When $C = 0$, $\Delta A_0 = 0$, $\Delta A_1 = 0$, $\Delta B = 0$, and the matrices Z_{31}, Z_{32}, Z_{33} are zeros, the matrix Z can be reduced to $Z = \begin{pmatrix} Z_{11} & Z_{21}^{\mathrm{T}} \\ Z_{21} & Z_{22} \end{pmatrix}$. Let $\widetilde{Z} = \begin{pmatrix} \widetilde{Z}_{11} & \widetilde{Z}_{21}^{\mathrm{T}} \\ \widetilde{Z}_{21} & \widetilde{Z}_{22} \end{pmatrix} = X^{\mathrm{T}} Z X$. Then, following the same steps as in the proof of Theorem 1, we obtain the following corollary.

Corollary 1. Consider the nominal system of (1)–(4). For given $\varepsilon_1, \varepsilon_2 \in \mathbf{R}$, if there exist $\overline{Q} = \overline{Q}^{\mathrm{T}} > 0$, $\overline{R} = \overline{R}^{\mathrm{T}} > 0$, $\overline{W} = \overline{W}^{\mathrm{T}} > 0$, $X_1 = X_1^{\mathrm{T}} > 0$, X_2, $X_3 \in \mathbf{R}^{n\times n}$, $U, G \in \mathbf{R}^{m\times n}$, and positive scalars β, δ, satisfying (13) and the following LMIs:

$$\begin{pmatrix} \Omega_{11} & * & * & * & * \\ \Omega_{21(j)} & \Omega_{22} & * & * & * \\ -\varepsilon_1 \overline{Q} A_1^{\mathrm{T}} & (1-\varepsilon_2)\overline{Q} A_1^{\mathrm{T}} & -(1-d_1)\overline{Q} & * & * \\ h_m X_2 & h_m X_3 & 0 & -h_m \overline{R} & * \\ X_1 & 0 & 0 & 0 & -\overline{Q} \end{pmatrix} < 0$$

$$j = 1, \cdots, 2^m$$

$$\begin{pmatrix} \widetilde{Z}_{11} & * & * \\ \widetilde{Z}_{21} & \widetilde{Z}_{22} & * \\ \varepsilon_1 \overline{R} A_1^{\mathrm{T}} & \varepsilon_2 \overline{R} A_1^{\mathrm{T}} & \overline{R} \end{pmatrix} \geqslant 0$$

where

$$\begin{cases} \Omega_{11} = X_2 + X_2^{\mathrm{T}} + \varepsilon_1 (X_1 A_1^{\mathrm{T}} + A_1 X_1) + h_m \widetilde{Z}_{11} \\ \Omega_{21(j)} = X_3^{\mathrm{T}} - X_2 + (A_0 + \varepsilon_2 A_1) X_1 + \\ \qquad B(D_j U + \overline{D}_j G) + h_m \widetilde{Z}_{21} \\ \Omega_{22} = -X_3^{\mathrm{T}} - X_3 + h_m \widetilde{Z}_{22} \end{cases}$$

then the system is asymptotically stabilized by $K = UX_1^{-1}$ for any initial condition in Ξ_δ with

$$\delta^2 \max\{\lambda_{\max}(X_1^{-1}) + 2\frac{h_m}{(1-d_1)}\lambda_{\max}(\overline{Q}^{-1});$$

$$2h_m^2 \lambda_{\max}(\overline{Q}^{-1}) + h_m \lambda_{\max}(\overline{R}^{-1})\} \leqslant \beta^{-1}.$$

Remark 1. By Corollary 1, if we take $\varepsilon_1 = 0$, our result reduces to Theorem 1 in [4, 12]. Clearly, $\varepsilon_1 = 0$ is not the best choice. This implies that our result is less conservative than that of [4, 12]. This is an advantage of our result since generally a comparison between results in terms of LMIs is made by numerical examples while in this paper it is established theoretically that our result is less conservative than those of [4, 12].

Remark 2. The global stability cannot be ensured in general. Furthermore, in general, when it is possible to compute a global stabilizing control law[13], it is difficult to simultaneously guarantee good performance and robustness for the closed-loop system.

Theorem 1 provides a condition allowing us to compute both a control law and a domain of attraction in which the closed loop neutral system is robustly stable. It is interesting to come up with a solution such that the domain of initial conditions is the largest possible. However, from the nonlinearity of (14), this is very difficult or even impossible. Assume the following conditions:

$$X_1^{-1} \leqslant \sigma_1 I, \ \ \overline{Q}^{-1} \leqslant \sigma_2 I, \ \ \overline{R}^{-1} \leqslant \sigma_3 I, \ \ \overline{W}^{-1} \leqslant \sigma_4 I.$$

By Schur complement, the following LMIs are obtained:

$$\begin{pmatrix} \sigma_1 I & I \\ I & X_1 \end{pmatrix} \geqslant 0, \ \begin{pmatrix} \sigma_2 I & I \\ I & \overline{Q} \end{pmatrix} \geqslant 0, \tag{19}$$

$$\begin{pmatrix} \sigma_3 I & I \\ I & \overline{R} \end{pmatrix} \geqslant 0, \ \begin{pmatrix} \sigma_4 I & I \\ I & \overline{W} \end{pmatrix} \geqslant 0. \tag{20}$$

It follows that condition (14) is satisfied if

$$\delta^2 \max\{\sigma_1 + 2\frac{h_m}{(1-d_1)}\sigma_2; \ 2h_m^2 \sigma_2 + h_m \sigma_3 + \frac{1}{(1-d_2)}\sigma_4\} \leqslant \beta^{-1}$$

holds.

Combining the facts derived above, we can construct a feasibility problem for given h_m as follows:

$$\text{Find} \quad \overline{Q}, \overline{R}, \overline{W}, X_1, X_2, X_3, U, G, \Lambda, \beta, \varepsilon_1, \varepsilon_2, \delta, \sigma_i, \quad i = 1, \cdots, 4$$

$$\text{subject to} \ \ X_1 > 0, \overline{Q} > 0, \overline{R} > 0, \overline{W} > 0, \Lambda > 0, \beta > 0, \delta > 0, \sigma_i > 0, i = 1, \cdots, 4, \text{and (12)–(14), (19), (20).} \tag{21}$$

Given h_m, if the above problem has a solution, we say that there exists a controller $u(t) = UX_1^{-1}x(t)$ that guarantees stability of the saturated neutral system (1)–(4).

Remark 3. In the derivation of Theorem 1, two tuning parameters ε_1 and ε_2 are introduced. An interesting question is how to find the desired values of these parameters. In this case, the desired values for ε_1 and ε_2 obtained correspond to the largest bound of time delay for which it is possible to find a feasible solution. A way to overcome the computational difficulties of solving this problem consists in considering the following algorithm:

Algorithm 1.

Step 1. Let $\beta = 1$ and fix h_{m0}, h_{mstep}, ε_{10}, ε_{20}, δ_0 small enough to have a feasible solution $\overline{Q}$, $\overline{R}$, $\overline{W}$, X_1, X_2, X_3, U, G, Λ, σ_i, $i = 1, \cdots, 4$ for (21).

Step 2. Let $h_m = h_{m0} + h_{mstep}$, $\varepsilon_1 = \varepsilon_{10}$, $\varepsilon_2 = \varepsilon_{20}$, and solve (21).

Step 3. If (21) is feasible, let $h_{m0} = h_m$, $\varepsilon_{10} = \varepsilon_1$, $\varepsilon_{20} = \varepsilon_2$, and go to Step 2. Otherwise, $h_m = h_{m0} - h_{mstep}$.

Step 4. If ε_1 and ε_2 are sufficiently large go to Step 5, otherwise change ε_1 and ε_2, and go to Step 3.

Step 5. Stop, the desired values of ε_1 and ε_2 are ε_{10} and ε_{20}, respectively.

4 Numerical examples

In this section, we provide three numerical examples to demonstrate that the proposed method gives less conservative results than the existing ones.

Example 1. Consider the nominal neutral system provided in [31] in the form of (1) with $B = 0$, $\Delta A_0(t) = 0$, $\Delta A_1(t) = 0$, $\Delta B(t) = 0$, $d_1 = 0$, $d_2 = 0$, and

$$A_0 = \begin{pmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{pmatrix}, \ \ A_1 = \begin{pmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{pmatrix},$$

$$C = \begin{pmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{pmatrix}.$$

The results are compared in Table 1. It can be seen that the delay-dependent stability condition of Lemma 3 is less conservative in the sense that the computed maximum delay bound is larger.

Table 1 Comparison of maximal delay bounds of h_m

Methods	h_m
Lien et al.[31]	0.3
Chen et al.[17]	0.5658
Fridman[19]	0.74
Lien and Chen[20]	0.8844
Park and Kwon[21]	1.3718
Yang et al.[23]	1.533
Chen[16]	1.5497
Lemma 3	1.7191

Example 2. We consider a state-feedback example, taken from [4, 7] ($C = 0, \Delta A_0(t) = 0, \Delta A_1(t) = 0, \Delta B(t) = 0$) where

$$A_0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \ A_1 = \begin{pmatrix} -1 & -1 \\ 0 & 0.9 \end{pmatrix}, \ B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Now, we address the problem of finding a state-feedback controller for guaranteeing stability of the above system. Applying Corollary 1 of this paper, we take $\varepsilon_1 = 0.36$, $\varepsilon_2 = 0.43$, $d_1 = 0$, and $d_2 = 0$. Table 2 gives a comparison of several results of the maximum allowable bound of delay and corresponding control gain.

Table 2 Stability bound of h_m and control gain K

Methods	h_m	Control gain
Li and Souza[7]	$h_m \leqslant 0.999$	$K = -(0.10452 \ \ 749058)$
Fridman and Shaked[4]	$h_m \leqslant 1.51$	$K = -(58.31 \ \ 294.935)$
Cho et al.[3]	$h_m \leqslant 1.6$	$K = -(0.001 \ \ 1.0154)$
Theorem 3 in this paper	$h_m \leqslant 1.8214$	$K = -(0.3670 \ \ 1.3124) \times 10^4$

Example 3. Consider the example given in [11]. The system is described by (1) with $C = 0$, $\Delta A_0(t) = 0$, $\Delta A_1(t) = 0$, $\Delta B(t) = 0$ and

$$A_0 = \begin{pmatrix} 0.5 & -1 \\ 0.5 & -0.5 \end{pmatrix}, \ A_1 = \begin{pmatrix} 0.6 & 0.4 \\ 0 & -0.5 \end{pmatrix},$$

$$B = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \ \bar{u} = 5.$$

When $d_1 = 0$, the delay is time-invariant. In [32], for $A_{11} = \begin{pmatrix} 0.55 & 0.6 \\ 0 & -0.2 \end{pmatrix}$, the upper bound on the time delay was found to be $h_m = 0.0332$. In [11], stabilization by a saturated memoryless state feedback law was accomplished for $h_m \leqslant 0.35$ with a maximum radius of the stability ball of 0.9680.

Fridman et al.[12] gave a bound of $h_m = 1.854$ according to Theorem 1 therein, while by using Matlab LMI Toolbox we obtain $h_m = 1.7543$, with a stability radius $\delta = 0.1666$.

By Theorem 1, for $d_1 = 0$, $d_2 = 0$, $\varepsilon_1 = 0.11$, $\varepsilon_2 = 0.88$, $\beta = 1$, and $h_m \leqslant 1.89$, the closed-loop system is found to be asymptotically stable with the stabilizing gain $K = -(5.3385 \ \ 0.8452)$ and the stability ball radius $\delta = 0.119$.

For $\varepsilon_1 = 0.46$, $\varepsilon_2 = 0.82$, and $\beta = 1$, the upper bound on the time delay is found to be $h_m \leqslant 2$. For $h_m = 2$, the stabilizing gain is $K = -(5.7702 \ \ 0.9754)$, with a stability ball radius of $\delta = 0.0718$.

Table 3 gives a comparison of stability ball radii for various constant delay values. From Table 3, we find that δ increases when the system time delay h_m decreases, and our approach gives larger estimation of the domain of attraction.

By numerical simulation, we show in Fig. 1 the trajectories of the saturated closed-loop system and the domain of attraction for the state trajectories for $h_m = 0.35$. The outer ellipsoid is D_e and the inner ellipsoid has a circle radius of $\delta = 2.9089$.

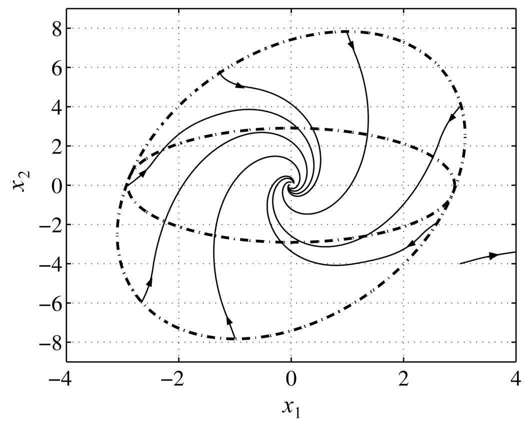

Fig. 1 Trajectories and estimate of the domain of attraction for $h_m = 0.35$

Table 3 Comparison of stability ball radii δ

Methods	$\delta = 0.35$	$\delta = 1$	$\delta = 1.7543$	$\delta = 1.8$	$\delta = 1.89$
Proposed method $\varepsilon_1 = 0.11, \varepsilon_2 = 0.88$	2.9089	1.5001	0.4142	0.3514	0.119
Fridman et al.[12] $\varepsilon_1 = 0, \ \varepsilon_2 = 0.95$	2.8386	1.4513	0.1666	Infeasibility	
Cao et al.[11]	0.9680	Infeasibility			

5 Conclusions

In this paper, we have presented several new delay-dependent robust stabilization conditions for neutral systems with saturating actuators. The method is based on a transformation of the actuator saturation nonlinearities into a convex combination of polytope, Lyapunov-Krasovskii functional, and the free weighting matrices technique. A state feedback control law governing the stability of the system against perturbations is constructed by solving LMIs depending on two tuning parameters. To overcome the difficulty in finding the best values of the tuning parameters that give the largest bound of the delay $h(t)$, an iterative algorithm is proposed. An estimation of the domain of attraction is also proposed. It is also shown that our result is more general and less conservative than those of [4, 12]. Numerical examples have shown that the approach of this paper gives much less conservative results than the existing ones in literature.

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