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A Minimum Approximate-BER Beamforming Approach for PSK Modulated Wireless Systems

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Abstract: A beamforming algorithm is introduced based on the general objective function that approximates the bit error rate for the wireless systems with binary phase shift keying and quadrature phase shift keying modulation schemes. The proposed minimum approximate bit error rate (ABER) beamforming approach does not rely on the Gaussian assumption of the channel noise. Therefore, this approach is also applicable when the channel noise is non-Gaussian. The simulation results show that the proposed minimum ABER solution improves the standard minimum mean squares error beamforming solution, in terms of a smaller achievable system′s bit error rate.

Keywords: Adaptive beamforming, bit error rate, gradient descent, smart antenna.

1 Introduction

The demand for increasing the capacity of mobile communication systems has motivated the need for new space-division multiple-access technologies to further improve the efficiency of spectral utilisation. Spatial processing with adaptive antenna arrays is an attractive option as it offers to increase the capacity substantially by exploiting the spatial dimension[1−8]. Adaptive beamforming[2,6] is capable of recovering a set of signals that are transmitted with the same carrier frequency, provided that they are separated in the spatial or angular domain. Signals received by the multiple elements in an antenna array are combined by the beamforming process via appropriating weighting parameters, and the values of the beamformer′s weights are obtained based on some chosen optimisation criterion. The classical beamforming design is the standard minimum mean squares error (MMSE) solution[9], in which the mean square error (MSE) between the desired and actual array outputs are minimised. However, for the communication systems, the ultimate performance indicator is the system′s bit error rate (BER), not the MSE. This has motivated researches in the design of adaptive minimum bit error rate (MBER) filters[10] with a variety of applications, including channel equalisation[11,12] and adaptive beamforming[13,14].

Note that the beamforming process effectively solves a linear classification problem with the classification boundary determined by the beamforming weight vector. The analytical MBER solution in [13, 14] is derived under the assumption of additive white Gaussian noise. In practice, the probability distribution of the channel noise may be either unknown or non-Gaussian, and it may not be easy to generalise the optimal MBER solution in [13, 14] to the case of non-Gaussian channel noise. This contribution proposes a new adaptive beamforming design by minimising the cost function that is an approximate BER (ABER). A significant advantage of the proposed minimum ABER beamforming design is that it does not require the Gaussian assumption of the channel noise. In fact, the proposed approach does not need to know the probability distribution of the channel noise. We apply this minimum ABER beamforming design to both the binary phase shift keying (BPSK) and quadrature phase shift keying (QPSK) modulated wireless systems. Our simulation results obtained demonstrate that the proposed minimum ABER solution is able to improve significantly over the standard MMSE design, in terms of a smaller achievable BER.

2 System model

Consider a coherent wireless communication system that supports M users, where each user transmits on the same angular carrier frequency ω. Assume that the channel is non-dispersive and hence it does not induce intersymbol interference. The baseband transmitted signal of user i is given by

$$m_i(k) = A_i\, b_i(k),\ 1 \leqslant i \leqslant M \tag{1}$$

where the transmitted symbol $b_i(k) \in \{\pm 1\}$ for the BPSK system, or $b_i(k) \in \{\pm 1 \pm j\}$ for the QPSK system, and A_i denotes the complex-valued and non-dispersive channel coefficient for user i. Without loss of generality, the user 1 is assumed to be the desired user and all the other users are the interfering users. The receiver uses a uniformly-spaced linear antenna array with L elements, and the antenna array has an element spacing of half of the wavelength. The received signals at the the antenna array are represented by[2,6]

$$x_l(k) = \sum_{i=1}^{M} m_i(k) \mathrm{e}^{j\pi(l-1)\sin(\theta_i)} + n_l(k) \tag{2}$$

for $1 \leqslant l \leqslant L$, where θ_i is the direction of arrival for user i, and $n_l(k)$ is white noise, a complex-valued channel with zero mean, and $\mathrm{E}[|n_l(k)|^2] = 2\sigma_n^2$. The received signal vector $\boldsymbol{x}(k) = [x_1(k)\ x_2(k) \cdots x_L(k)]^{\mathrm{T}}$ can be expressed as

$$\boldsymbol{x}(k) = \boldsymbol{P}\boldsymbol{b}(k) + \boldsymbol{n}(k) \tag{3}$$

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where the transmitted symbol vector $\boldsymbol{b}(k) = [b_1(k)\ b_2(k) \cdots b_M(k)]^{\mathrm{T}}$, the channel noise vector $\boldsymbol{n}(k) = [n_1(k)\ n_2(k) \cdots n_L(k)]^{\mathrm{T}}$, and the system matrix $\boldsymbol{P} = [A_1\boldsymbol{s}_1\ A_2\boldsymbol{s}_2 \cdots A_M\boldsymbol{s}_M]$ with the associated steering vectors $\boldsymbol{s}_i = [1\ \mathrm{e}^{j\pi\sin(\theta_i)} \cdots \mathrm{e}^{j\pi(L-1)\sin(\theta_i)}]^{\mathrm{T}}$. Define the desired-user signal to noise ratio (SNR) as SNR$= E_s|A_1|^2/2\sigma_n^2$, where E_s is the average symbol energy, and the desired-user signal to interferer i ratio (SIR) as SIR$_i = |A_1|^2/|A_i|^2$ for $2 \leqslant i \leqslant M$.

The beamformer output is given by

$$y(k) = \boldsymbol{w}^{\mathrm{H}}\boldsymbol{x}(k) \tag{4}$$

where $\boldsymbol{w} = [w_1\ w_2 \cdots w_L]^{\mathrm{T}}$ is the complex-valued beamformer weight vector. Using the signal model (3), the beamformer output can be expressed equivalently as

$$y(k) = \boldsymbol{w}^{\mathrm{H}}\boldsymbol{P}\boldsymbol{b}(k) + e(k) \tag{5}$$

where $e(k)$ is the complex-valued noise at the beamformer's output having zero mean and $\mathrm{E}[|e(k)|^2] = 2\sigma_n^2\boldsymbol{w}^{\mathrm{H}}\boldsymbol{w}$. The estimate for the transmitted symbol $b_1(k)$ is

$$\hat{b}_1(k) = \mathrm{sgn}(\mathrm{Re}[y(k)]) \tag{6}$$

for the BPSK system and

$$\hat{b}_1(k) = \mathrm{sgn}(\mathrm{Re}[y(k)]) + j\,\mathrm{sgn}(\mathrm{Im}[y(k)]) \tag{7}$$

for the QPSK system, where Re[·] and Im[·] denote the real and imaginary parts, respectively, and

$$\mathrm{sgn}(r) = \begin{cases} +1, & r \geqslant 0 \\ -1, & r < 0. \end{cases} \tag{8}$$

The classical MMSE beamforming solution is defined by

$$\boldsymbol{w}_{\mathrm{MMSE}} = \left(\boldsymbol{P}\boldsymbol{P}^{\mathrm{H}} + \frac{2\sigma_n^2}{E_s}\boldsymbol{I}_L\right)^{-1}\boldsymbol{p}_1 \tag{9}$$

where $\boldsymbol{p}_1$ denotes the first column of $\boldsymbol{P}$, and $\boldsymbol{I}_L$ is the $L \times L$ identity matrix. The MMSE solution is computationally attractive as it is given in a closed form. However, in general, the MMSE design is not the optimal MBER design. If the channel noise $\boldsymbol{n}(k)$ is Gaussian distributed, the MBER solution $\boldsymbol{w}_{\mathrm{MBER}}$ for the beamforming (4) can been derived[13,14]. The main contribution of the current work is to design an approximate MBER beamforming solution when the probability distribution of $\boldsymbol{n}(k)$ is non-Gaussian or even unknown.

3 The minimum ABER design for BPSK systems

Consider initially the case of BPSK systems. Since any positive scaling of $\boldsymbol{w}$ does not change the decision (6), we may apply the constraint $\boldsymbol{w}^{\mathrm{H}}\boldsymbol{w} = 1$ and parameterise the weight vector $\boldsymbol{w}$ by the real-valued parameter vector $\boldsymbol{\alpha} = [\alpha_1\ \alpha_2\ \cdots\ \alpha_L]^{\mathrm{T}}$:

$$\boldsymbol{w} = [s_1\mathrm{e}^{-j\alpha_1}\ s_2\mathrm{e}^{-j\alpha_2}\ \cdots\ s_L\mathrm{e}^{-j\alpha_L}]^{\mathrm{T}} \tag{10}$$

where $s_l = |w_{\mathrm{MMSE},l}|/\|\boldsymbol{w}_{\mathrm{MMSE}}\|$ and $w_{\mathrm{MMSE},l}$ denotes the l-th element of $\boldsymbol{w}_{\mathrm{MMSE}}$.

3.1 The approximate bit error rate

Because only the real part of $y(k)$ is used for signal detection, let us consider the real part $y_R(k) = \mathrm{Re}[y(k)]$ of the beamformer output

$$\begin{aligned} y_R(k) &= \sum_{i=1}^{M} A_i b_i(k)\left(\sum_{l=1}^{L} s_l\cos\left[\pi(l-1)\sin(\theta_i) + \alpha_l\right]\right) + \\ \varepsilon(k) &= \bar{y}_R(k) + \varepsilon(k) \end{aligned} \tag{11}$$

where $\varepsilon(k) = \mathrm{Re}[e(k)]$, having zero mean and variance σ_n^2. Assume that all the users are symbol-synchronised and a training data set $\{\boldsymbol{b}(k)\}_{k=1}^{N}$ is given. The aim is to find the solution of $\boldsymbol{\alpha}$ such that the BER

$$J = \frac{1}{N}\sum_{k=1}^{N} Id[b_1(k), \bar{y}_R(k)] \tag{12}$$

is minimised, where the indicator function is defined as

$$Id[b_1(k), r] = \begin{cases} 1, & \text{if } b_1(k) \neq \mathrm{sgn}(r) \\ 0, & \text{if } b_1(k) = \mathrm{sgn}(r). \end{cases} \tag{13}$$

Unfortunately, the true BER given by (12) is not parameterised as a differentiable function of the unknown parameters $\boldsymbol{\alpha}$ for the use of functional optimisation. In order to overcome this problem, assuming that $\mathrm{Prob}(\mathrm{sgn}(\bar{y}_R(k)) \neq \mathrm{sgn}(y_R(k))|b_1(k) = +1) = \mathrm{Prob}(\mathrm{sgn}(\bar{y}_R(k)) \neq \mathrm{sgn}(y_R(k))|b_1(k) = -1)$, which is valid as the BPSK symbol constellation is symmetric, J can be alternatively represented as

$$\begin{aligned} J &= \frac{1}{2N}\sum_{k=1}^{N}(1 - b_1(k)\mathrm{sgn}(\bar{y}_R(k))) \approx \\ &\frac{1}{2N}\sum_{k=1}^{N}(1 - b_1(k)\tanh(\gamma\bar{y}_R(k))) = J_A(\boldsymbol{\alpha}) \end{aligned} \tag{14}$$

where $J_A(\boldsymbol{\alpha})$ is referred to as the ABER loss function, and $\gamma > 0$ is a scalar. Note that J_A becomes a differentiable loss function, becauce nonlinear optimisation techniques can be applied to derive the minimum ABER solution.

The minimum ABER solution is defined as the solution of

$$\frac{\partial}{\partial\boldsymbol{\alpha}}J_A = 0 \tag{15}$$

which can be obtained numerically, for example, using the following normalised gradient descent algorithm. Specifically, the MMSE weight vector solution $\boldsymbol{w}_{\mathrm{MMSE}}$ is used to initialise the parameter vector $\boldsymbol{\alpha}$ according to

$$\alpha_l(0) = -\arctan(\mathrm{Im}[w_{\mathrm{MMSE},l}], \mathrm{Re}[w_{\mathrm{MMSE},l}]) \tag{16}$$

for $1 \leqslant l \leqslant L$, where $-\pi < \arctan(\mathrm{Im}[u], \mathrm{Re}[u]) < \pi$ gives the four quadrant arctangent value of a complex-valued number u. Then, the iterative normalised gradient descent algorithm is given by

$$\boldsymbol{\alpha}(\iota) = \boldsymbol{\alpha}(\iota - 1) - \eta\left.\frac{\dfrac{\partial}{\partial\boldsymbol{\alpha}}J_A}{\left\|\dfrac{\partial}{\partial\boldsymbol{\alpha}}J_A\right\|}\right|_{\boldsymbol{\alpha}=\boldsymbol{\alpha}(\iota-1)} \tag{17}$$

where η is the learning rate and ι indicates the iteration index. The gradient vector $\frac{\partial}{\partial \boldsymbol{\alpha}} J_A = \left[\frac{\partial}{\partial \alpha_1} J_A \ \frac{\partial}{\partial \alpha_2} J_A \cdots \frac{\partial}{\partial \alpha_L} J_A\right]^{\mathrm{T}}$ with

$$\frac{\partial}{\partial \alpha_l} J_A = \frac{\gamma}{2N} \sum_{k=1}^{N} b_1(k) \left(1 - (\tanh(\gamma \bar{y}_R(k)))^2\right) \cdot \left(\sum_{i=1}^{M} A_i b_i(k) s_l \sin\left[\pi(l-1)\sin(\theta_i) + \alpha_l\right]\right) \quad (18)$$

for $1 \leqslant l \leqslant L$. It is easy to verify that J_A can be made arbitrarily close to the true BER by increasing γ. However, in order for the above formula not to produce zero values so that the optimisation procedure does take place, γ should not be set to a value too large, e.g., $\gamma < 50$.

The optimal MBER beamformer given in [13] is derived under the condition that the channel noise is Gaussian distributed. A key difference between the proposed minimum ABER design and the MBER solution in [13] is that no assumption is made regarding the probability distribution of the channel noise. In fact, from the above derivation, it can be seen that the proposed approach does not need to know the probability distribution of the channel noise. This is advantageous in practice as the probability distribution of the channel noise may be non-Gaussian or even unknown.

3.2 Simulation study for BPSK systems

For the BPSK system, the total number of all the legitimate transmitted sequences of $\boldsymbol{b}(k)$ is $K = 2^M$. Under the condition that the channel noise $\boldsymbol{n}(k)$ is Gaussian, the analytic formula for the true BER of the beamformer in (4) is given by[13]

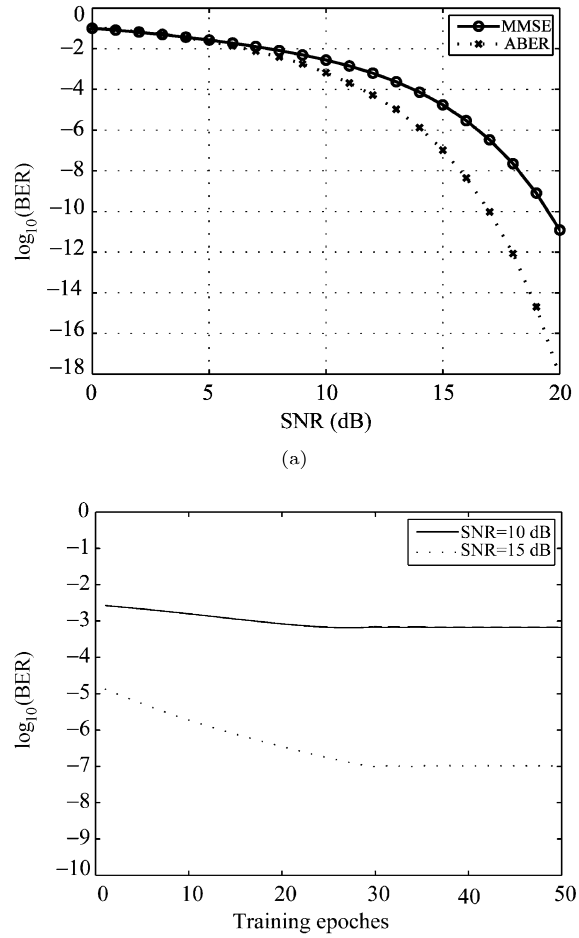
$$P_{E,\mathrm{BPSK}}(\boldsymbol{\alpha}) = \frac{2}{K} \sum_{k=1}^{K/2} Q\left(\frac{\mathrm{Re}[\boldsymbol{w}(\boldsymbol{\alpha})^{\mathrm{H}} \boldsymbol{P} \boldsymbol{b}(k)]}{\sigma_n}\right) \quad (19)$$

where the Q-function is defined by

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \mathrm{e}^{-\frac{r^2}{2}} \mathrm{d}r \quad (20)$$

and the values of $\boldsymbol{b}(k)$ (without repetition) which are determined by the half set of all the possible transmitted sequences if $b_1(k) = 1$. The true BER $P_{E,\mathrm{BPSK}}(\boldsymbol{\alpha})$ of (19) is used as the performance indicator in the following experiment, where the Gaussian channel noise is used.

Consider the system with a two-element antenna array that supports 5 BPSK users ($M = 5$). The two elements of the array are separated by the half of the wavelength and the angular positions of the five users are given by 15°, −30°, 60°, 80°, and −70°, respectively. Initially, all the five users have an equal power, and therefore, $\mathrm{SIR}_i = 0\,\mathrm{dB}$ for $2 \leqslant i \leqslant 5$. For each SNR value, a set of 200 training data points is generated for testing the proposed minimum ABER algorithm in comparison with the MMSE solution. Fig. 1 (a) demonstrates that the minimum ABER design is superior over the standard MMSE solution. The evolution of the BER over the training epochs in Fig. 1 (b) shows that the excellent convergence speed of the normalised gradient descent algorithm is employed to arrive at the minimum ABER solution. Extensive experiments have been conducted to investigate the influence of γ to the algorithmic performance, and the results show that the similar results are obtained when $5 < \gamma < 20$.



(a)

(b)

Fig. 1 Experiment results for the BPSK system with equal user power. (a) BER performance comparison of the MMSE and minimum ABER beamforming solutions; (b) Convergence rate of the normalised gradient descent algorithm. The learning rate was set to η=0.02 and γ=8.

In the second experiment, all the four interfering users have more power (6 dB) than one desired user. Therefore, $\mathrm{SIR}_i = -6\,\mathrm{dB}$ for $2 \leqslant i \leqslant 5$. Fig. 2 shows the superior performance of the minimum ABER beamforming design over the traditional MMSE solution under this high interference condition.

4 The minimum ABER design for QPSK systems

This section extends the minimum ABER approach to the QPSK system. The beamforming for QPSK signals is equivalent to solving a 4-class classification problem. Alternatively, the problem can be decomposed into the two 2-class classifiers in parallel.

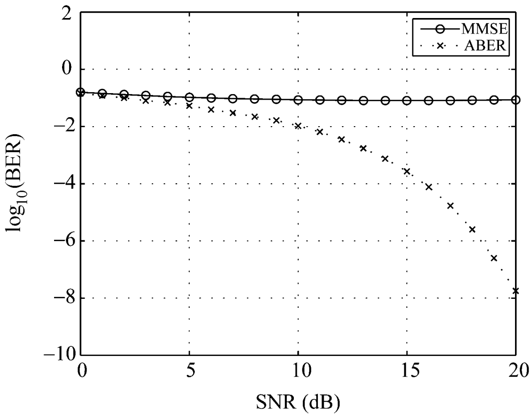


Fig. 2 Experiment results for the BPSK system with high interference: BER performance comparison of the MMSE and minimum ABER beamforming solutions. The learning rate was set to $\eta = 0.02$ and $\gamma = 8$.

4.1 A dual beamformer model

The traditional beamforming employs a single linear spatial filter (4), and the optimal MBER solution based on this linear beamformer is given in [14] under the assumption of Gaussian channel noise. We adopt a new dual beamformer model, in which the outputs of the two spatial filters are given by

$$y_w(k) = \boldsymbol{w}^{\mathrm{H}}\boldsymbol{x}(k) \tag{21}$$

and

$$y_u(k) = \boldsymbol{u}^{\mathrm{H}}\boldsymbol{x}(k) \tag{22}$$

respectively, where both $\boldsymbol{w} = [w_1\ w_2 \cdots w_L]^{\mathrm{T}}$ and $\boldsymbol{u} = [u_1\ u_2 \cdots u_L]^{\mathrm{T}}$ are the complex-valued weight vectors. Instead of using decision (7), the estimate of the transmitted QPSK symbol $b_1(k)$ is given by

$$\hat{b}_1(k) = \mathrm{sgn}(\mathrm{Re}[y_w(k)]) + j\,\mathrm{sgn}(\mathrm{Im}[y_u(k)]). \tag{23}$$

Note that this dual filtering model with the (nonlinear) decision (23) is referred to as the almost linear filtering in the literature [15–17]. It is also well-known that, due to the circular distribution of the channel noise, the MMSE solution based on this duel filtering model is identical to the MMSE solution derived from the single filtering model (4). The motivation of using the dual beamforming model here is that it makes the extension of the minimum ABER design for the BPSK system to the QPSK system easier.

4.2 The approximate bit error rate

Similarly to the BPSK case, the constraints $\boldsymbol{w}^{\mathrm{H}}\boldsymbol{w} = 1$ and $\boldsymbol{u}^{\mathrm{H}}\boldsymbol{u} = 1$ are enforced by the parameterisation of the two real-valued parameter vectors $\boldsymbol{\alpha} = [\alpha_1\ \alpha_2 \cdots \alpha_L]^{\mathrm{T}}$ and $\boldsymbol{\beta} = [\beta_1\ \beta_2 \cdots \beta_L]^{\mathrm{T}}$, defined by (10) and

$$\boldsymbol{u} = [s_1 \mathrm{e}^{-j\beta_1}\ s_2 \mathrm{e}^{-j\beta_2} \cdots s_L \mathrm{e}^{-j\beta_L}]^{\mathrm{T}} \tag{24}$$

respectively. Denote $y_{w,R}(k) = \mathrm{Re}[y_w(k)]$ and $y_{u,I}(k) = \mathrm{Im}[y_u(k)]$. Then, from (21) and (22)

$$\begin{aligned} y_{w,R}(k) &= \sqrt{2}\sum_{i=1}^{M} A_i \left(\sum_{l=1}^{L} s_l f_i(\alpha_l)\right) + \varepsilon_R(k) = \\ &\quad \bar{y}_{w,R}(k) + \varepsilon_R(k) \end{aligned} \tag{25}$$

with

$$\begin{aligned} f_i(\alpha_l) &= \cos[\pi(l-1)\sin(\theta_i) + \alpha_l + \\ &\quad \arctan(\mathrm{Im}[b_i(k)], \mathrm{Re}[b_i(k)])] \end{aligned} \tag{26}$$

and

$$\begin{aligned} y_{u,I}(k) &= \sqrt{2}\sum_{i=1}^{M} A_i \left(\sum_{l=1}^{L} s_l g_i(\beta_l)\right) + \varepsilon_I(k) = \\ &\quad \bar{y}_{u,I}(k) + \varepsilon_I(k) \end{aligned} \tag{27}$$

with

$$\begin{aligned} g_i(\beta_l) &= \sin[\pi(l-1)\sin(\theta_i) + \beta_l + \\ &\quad \arctan(\mathrm{Im}[b_i(k)], \mathrm{Re}[b_i(k)])] \end{aligned} \tag{28}$$

where both $\varepsilon_R(k)$ and $\varepsilon_I(k)$ have zero mean and variance σ_n^2.

Also, assume that a block of the training data samples $\{\boldsymbol{b}(k)\}_{k=1}^{N}$ is available. The ABER loss function $J_A(\boldsymbol{\alpha}, \boldsymbol{\beta})$ is given by

$$J_A = \frac{1}{2}\left(J_{A,R}(\boldsymbol{\alpha}) + J_{A,I}(\boldsymbol{\beta})\right) \tag{29}$$

with

$$J_{A,R} = \frac{1}{2N}\sum_{k=1}^{N}\left(1 - \mathrm{Re}[b_1(k)]\tanh(\gamma \bar{y}_{w,R}(k))\right) \tag{30}$$

and

$$J_{A,I} = \frac{1}{2N}\sum_{k=1}^{N}\left(1 - \mathrm{Im}[b_1(k)]\tanh(\gamma \bar{y}_{u,I}(k))\right). \tag{31}$$

Because $\frac{\partial}{\partial \alpha_l} J_A = \frac{\partial}{\partial \alpha_l} J_{A,R}$ and $\frac{\partial}{\partial \beta_l} J_A = \frac{\partial}{\partial \beta_l} J_{A,I}$, the minimisation of J_A can be decomposed into the two problems of minimising $J_{A,R}$ and $J_{A,I}$ in parallel.

Specifically, the minimum ABER solutions for $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ which define the spatial filters (21) and (22) can be obtained numerically using the following normalised gradient descent algorithm. Given the initial parameter vectors $\boldsymbol{\alpha}(0) = \boldsymbol{\beta}(0)$, with $\alpha_l(0) = \beta_l(0)$ for $1 \leqslant l \leqslant L$ as defined in (16). The parameter vector $\boldsymbol{\alpha}$ is updated according to

$$\boldsymbol{\alpha}(\iota) = \boldsymbol{\alpha}(\iota - 1) - \eta \left.\frac{\frac{\partial}{\partial \boldsymbol{\alpha}} J_{A,R}}{\left\|\frac{\partial}{\partial \boldsymbol{\alpha}} J_{A,R}\right\|}\right|_{\boldsymbol{\alpha} = \boldsymbol{\alpha}(\iota - 1)} \tag{32}$$

while the parameter vector $\boldsymbol{\beta}$ is updated according to

$$\boldsymbol{\beta}(\iota) = \boldsymbol{\beta}(\iota - 1) - \eta \left.\frac{\frac{\partial}{\partial \boldsymbol{\beta}} J_{A,I}}{\left\|\frac{\partial}{\partial \boldsymbol{\beta}} J_{A,I}\right\|}\right|_{\boldsymbol{\beta} = \boldsymbol{\beta}(\iota - 1)} \tag{33}$$

where $\frac{\partial}{\partial \boldsymbol{\alpha}} J_{A,R} = \left[\frac{\partial}{\partial \alpha_1} J_{A,R}\ \frac{\partial}{\partial \alpha_2} J_{A,R} \cdots \frac{\partial}{\partial \alpha_L} J_{A,R}\right]^{\mathrm{T}}$ with

$$\begin{aligned} \frac{\partial}{\partial \alpha_l} J_{A,R} &= \frac{\gamma}{\sqrt{2}N}\sum_{k=1}^{N} \mathrm{Re}[b_1(k)] \cdot \\ &\left(1 - (\tanh(\gamma \bar{y}_{w,R}(k)))^2\right)\left(\sum_{i=1}^{M} A_i s_l g_i(\alpha_l)\right) \end{aligned} \tag{34}$$

for $1 \leqslant l \leqslant L$, and the gradient vector $\frac{\partial}{\partial \boldsymbol{\beta}} J_{A,I} = \left[\frac{\partial}{\partial \beta_1} J_{A,I} \ \frac{\partial}{\partial \beta_2} J_{A,I} \cdots \frac{\partial}{\partial \beta_L} J_{A,I}\right]^{\mathrm{T}}$ with

$$\frac{\partial}{\partial \beta_l} J_{A,I} = -\frac{\gamma}{\sqrt{2}N} \sum_{k=1}^{N} \mathrm{Im}[b_1(k)] \cdot \left(1 - (\tanh(\gamma \bar{y}_{u,I}(k)))^2\right) \left(\sum_{i=1}^{M} A_i s_l f_i(\beta_l)\right) \quad (35)$$

for $1 \leqslant l \leqslant L$.

4.3 Simulation study for QPSK systems

For the QPSK system, the total number of all the legitimate transmitted sequences of $\boldsymbol{b}(k)$ is $K = 4^M$. Similar to the derivation of the true BER for the beamforming model in the form of (4) and (7) given in [14], under the condition that the channel noise is Gaussian, the analytic formula of the true BER for the dual beamforming model in the form of (21)–(23) is

$$P_{E,\mathrm{QPSK}}(\boldsymbol{\alpha}, \boldsymbol{\beta}) = \frac{4}{K} \sum_{k=1}^{K/4} \left(Q\left(\frac{\mathrm{Re}\left[\boldsymbol{w}(\boldsymbol{\alpha})^{\mathrm{H}} \boldsymbol{P} \boldsymbol{b}(k)\right]}{\sigma_n}\right) + Q\left(\frac{\mathrm{Im}\left[\boldsymbol{u}(\boldsymbol{\beta})^{\mathrm{H}} \boldsymbol{P} \boldsymbol{b}(k)\right]}{\sigma_n}\right)\right) \quad (36)$$

where the values of $\boldsymbol{b}(k)$ (without repetition) are determined by the quarter set of all the possible sequences of $\boldsymbol{b}(k)$ if $b_1(k) = 1 + j$. In the simulation, the channel noise has a Gaussian distribution. Therefore, the BER of (36) is used to indicate the performance of a beamformer in the following experiments.

Consider the system that employs a three-element antenna array to support 4 QPSK users ($M = 4$). The array element spacing is the half of the wavelength, and the angular positions of the four users are given by 15°, −30°, 45°, and −70°, respectively. In the first experiment, all the four users have equal power. Therefore, $\mathrm{SIR}_i = 0\,\mathrm{dB}$ for $2 \leqslant i \leqslant 4$. The training data set contains 600 points ($N = 600$). Fig. 3 (a) shows the comparision of the BER performance of the MMSE design with the proposed minimum ABER solution. Fig. 3 (b) shows the convergence speed of the normalised gradient descent algorithm employed to obtain the minimum ABER solution. In the second experiment, the three interfering users have 1.6 dB higher power than desired user one, given $\mathrm{SIR}_i = -1.6\,\mathrm{dB}$ for $2 \leqslant i \leqslant 4$. Fig. 4 demonstrates the superior performance of the proposed minimum ABER design over the conventional MMSE solution under the high interference condition. More experiments for $5 < \gamma < 20$ have been conducted, and they show that the results are not sensitive to the value of γ.

5 Conclusions

A minimum ABER beamforming design has been proposed for the BPSK and QPSK wireless systems. The proposed beamforming approach has an important practical advantage over the existing MBER design, as it does not require knowledge of the channel noise distribution. The simulation results have demonstrated that the proposed minimum ABER beamforming achieves much smaller BER than the standard MMSE solution. Further research is required to implement the proposed design with a stochastic gradient algorithm.

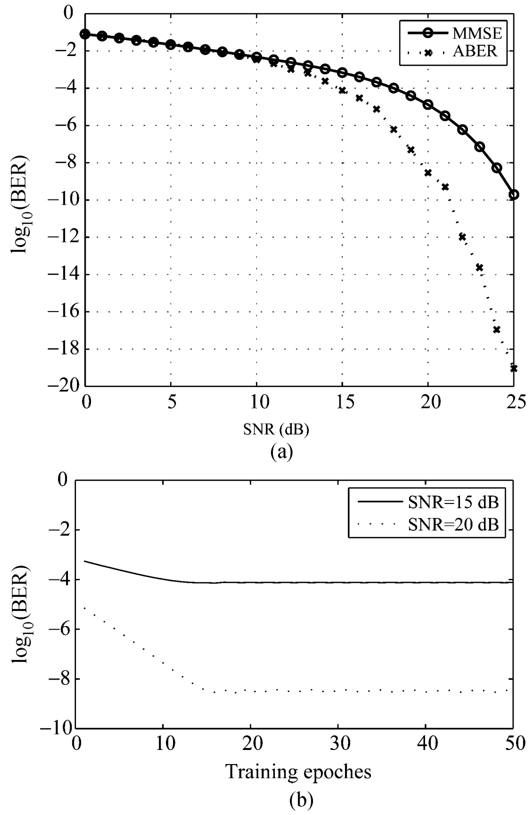


Fig. 3 Experiment results for the QPSK system with equal user power. (a) BER performance comparison of the MMSE and minimum ABER beamforming solutions; (b) Convergence rate of the normalised gradient descent algorithm. The learning rate was set to $\eta = 0.02$ and $\gamma = 8$.

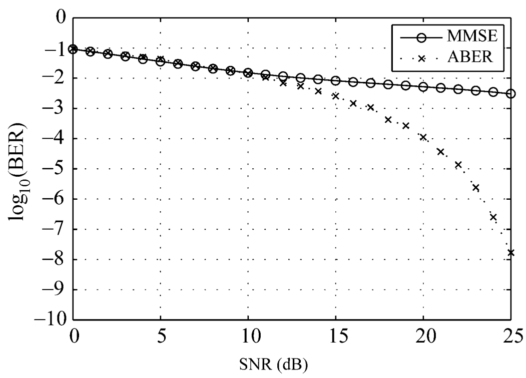


Fig. 4 Experimental results for the QPSK system with high interference: bit error rate performance comparison of the MMSE and minimum ABER beamforming solutions. The learning rate was set to $\eta = 0.02$ and $\gamma = 8$.

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